

Chapter Four REFRACTION OF LIGHT

Specific Objectives

By the end of this topic the learner should be able to:

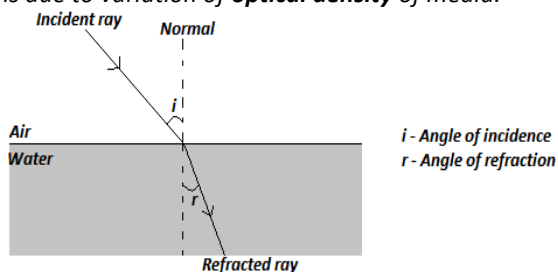
- a) Describe simple experiments to illustrate refraction of light
- b) State the laws of refraction of light
- c) Verify Snell's law
- d) Define refractive index
- e) Determine experimentally the refractive index
- f) Describe experiment to illustrate dispersion of white light
- g) Explain total internal reflection and its effects
- h) State the applications of total internal reflection
- i) Solve numerical problems involving refractive index and critical angle.

Content

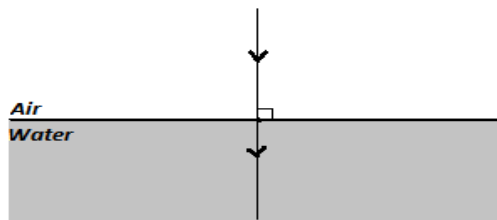
1. Refraction of light - laws of refraction (experimental treatment required)
2. Determination of refractive index
 - Snell's law
 - Real/ apparent depth
 - Critical angle
3. Dispersion of white light (Experimental treatment is required)
4. Total internal reflection and its effects: critical angle
5. Applications of total internal reflection
 - Prism periscope
 - Optical fibre
6. Problems on refractive index and critical angle

Definition of Refraction of Light

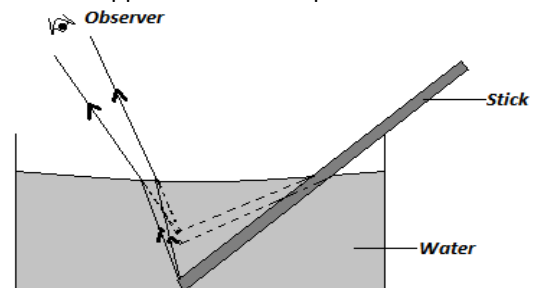
- Refraction of light refers to the change in direction of light at the interface as it travels from one medium to another at an angle, for example, a ray of light from air to water. The cause of refraction of light is the change in velocity of light as it travels from one medium to another. The change in velocity is due to variation of **optical density** of media.



- A ray that travels perpendicular to interface proceeds across the interface not deviated since the angle of incidence to the normal is zero.



- Refraction of light is the reason as to why;
 - a) a stick appears bent when part of it is in water

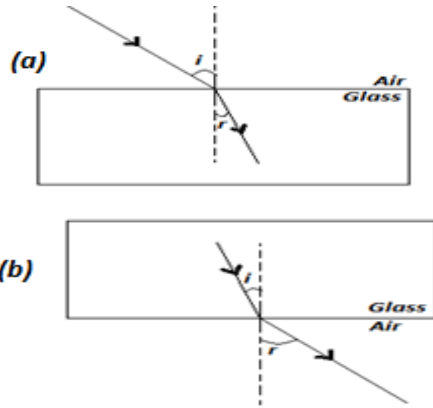


- b) a coin in a beaker of water appears near the surface than it actually is,
- c) a pool of water appears more shallow when viewed more obliquely etc. (Students to perform this practically)

Optical density (transmission density) and refraction of light

- A ray of light travelling from an optically less dense medium to an optically denser medium bends towards the normal after refraction e.g. a ray from air to glass block as in (a) below. The angle of incidence in this case is greater than angle of refraction.

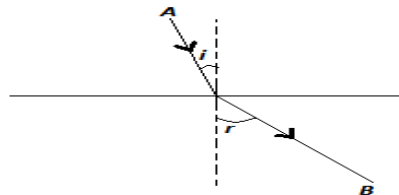
- A ray of light travelling from an optically dense medium to an optically less dense medium bends away from the normal after refraction e.g. a ray from glass to air as in (b) below. The angle of incidence in this case is less than angle of refraction.



- NB:** some media are physically denser but optically less dense than others e.g. kerosene is physically less dense but optically denser than water.

Exercise

- Define the term refraction
- Draw a diagram to show refraction for a ray of light across the following boundaries in the order they appear
 - air- water
 - water- glass
 - glass- air
 - glass- air- water
- The figure below shows how refraction occurs.



Which of the two media is optically denser? Explain.

- Explain with the help of a diagram why pencil placed partly in water appears bent.

Laws of Refraction

Law 1

- The incident ray, the refracted ray and the normal all lie in the same plane at the point of incidence.

Law 2 (Snell's law)

- It states that; "the ratio of the sine of the angle of incidence (i) to the sine of the angle of refraction (r) is a constant for a given pair of media".

Note:

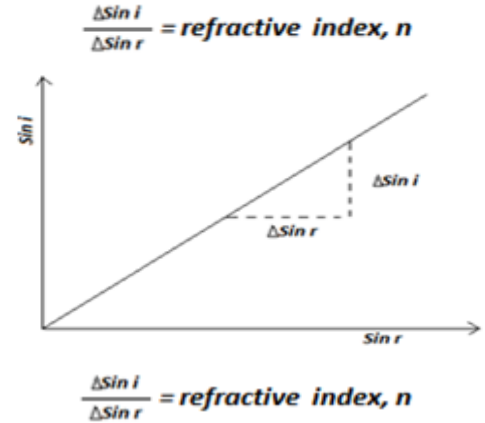
Absolute refractive index is the refractive index of a material with respect to vacuum. A vacuum has a refractive index of 1.000.

Examples

$$\frac{\sin i}{\sin r} = \text{constant.}$$

The constant is called **refractive index**.

- A graph of $\sin i$ against $\sin r$ is a straight line passing through the origin and slope of the graph gives the refractive index of a material.



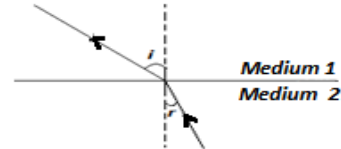
Refractive Index, n

- Refractive index is **defined as:** the ratio of the sine of the angle of incidence (i) to the sine of the angle of refraction (r) for a ray passing from one medium to another.

Consider a ray of light travelling from medium 1 to medium 2.

- For the two media, $\frac{\sin i}{\sin r} = \text{refractive index, } {}_1n_2$ (read as the refractive index of medium 2 with respect to medium 1)

- By the principle of reversibility of light, a ray travelling from medium 2 to medium 1 along the same path is refracted making the same angles.



$\frac{\sin r}{\sin i} = {}_2n_1$. r is the angle of incidence in this case.

but, $\frac{\sin i}{\sin r} = {}_1n_2 = \frac{1}{\frac{\sin r}{\sin i}}$

$${}_1n_2 = \frac{1}{{}_2n_1}$$

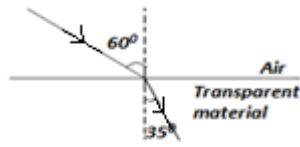
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but, $\frac{\sin i}{\sin r} = {}_1n_2 = \frac{1}{\frac{\sin r}{\sin i}}$

$${}_1n_2 = \frac{1}{{}_2n_1}$$

- Use the information given in the figures (a) and (b) below to calculate the refractive index ${}_a n_w$ and the angle θ

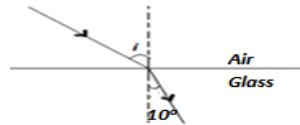
1. A ray of light passes through air into a certain transparent material. If the angles of incidence and refraction are 60° and 35° respectively, calculate the refractive index of the material
Solution



$$\text{Refractive index} = \frac{\sin i}{\sin r}$$

$$\text{Refractive index} = \frac{\sin 60}{\sin 35} = 1.510$$

2. Given that refractive index of glass is 1.5, calculate the angle of incidence for a ray of light travelling from air to glass if the angle of refraction is 10° .
Solution



$$\text{Refractive index} = \frac{\sin i}{\sin r}$$

$$1.5 = \frac{\sin i}{\sin 10^\circ}$$

$$\text{Angle of incidence, } i = \text{Sin}^{-1}(1.5 \times \sin 10^\circ) = 15.09^\circ$$

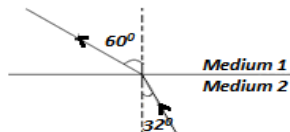
3. Calculate the refractive index for light travelling from glass to air given that ${}_a n_g = 1.572$.
Solution

$${}_g n_a = \frac{1}{{}_a n_g}$$

$${}_g n_a = \frac{1}{1.572} = 0.6361$$

Exercise

1. A ray of light striking a transparent material is refracted as shown below.

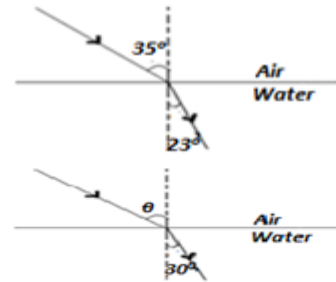


Calculate the refractive indices:

- a) ${}_1 n_2$
- b) ${}_2 n_1$

2. Calculate the angle of refraction for a ray of light striking an air glass interface making an angle of 50° with the interface (${}_a n_g = 1.526$)

Examples



4. State the principle of reversibility of light.

Refractive Index in Terms of Velocity

- Light travels faster in an optically less dense medium than in an optically denser medium. Consider a ray of light crossing the boundary from **medium 1** with **speed v_1** to **medium 2** with **speed v_2** , where v_1 is greater than v_2 as shown below.



- Refractive index, ${}_1 n_2$ of medium 2 with respect to medium 1 is given as:

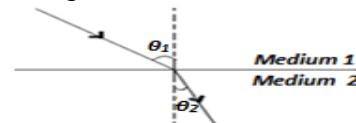
$${}_1 n_2 = \frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

$${}_1 n_2 = \frac{\text{velocity of lighth in medium1}}{\text{velocity of lighth in medium2}}$$

- Absolute refractive index n is the refractive index of the medium when light is travelling from the vacuum to the medium.

$$n = \frac{\text{Absolute refractive index, velocity of lighth in vacuum}}{\text{velocity of lighth in medium}}$$

- Consider the diagram below;



- If v_1 is the velocity of light in medium 1 of refractive index n_1 and v_2 the velocity of light in medium 2 of refractive index n_2 then;

$${}_1 n_2 = \frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1} = \frac{v_1}{v_2};$$

$$\Rightarrow n_1 \sin \theta_1 = n_2 \sin \theta_2;$$

$$\Rightarrow n \sin \theta = \text{constant}$$

- Therefore, the product of the refractive index of a medium and the angle a ray makes with the normal in the medium is a constant.

5. The figure below shows a glass prism of refractive index 1.5

1. Given that the refractive index of diamond is 2.51 and the velocity of light in air is $3.0 \times 10^8 \text{ ms}^{-1}$, calculate the velocity of light in diamond.

Solution

$$\begin{aligned} \text{refractive index of diamond} &= \frac{\text{velocity of lighth in air}}{\text{velocity of lighth in diamond}} \\ 2.51 &= \frac{3.0 \times 10^8 \text{ ms}^{-1}}{\text{velocity of lighth in diamond}}; \\ \Rightarrow \text{velocity of lighth in diamond} &= \frac{3.0 \times 10^8 \text{ ms}^{-1}}{2.51} \\ &= 1.195 \times 10^8 \text{ ms}^{-1} \end{aligned}$$

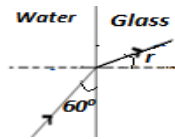
2. Given that the velocity of light in water is $2.27 \times 10^8 \text{ ms}^{-1}$ and in glass is $2.1 \times 10^8 \text{ ms}^{-1}$, calculate angle θ below.



Solution

$$\begin{aligned} \frac{\sin \theta_{\text{water}}}{\sin \theta_{\text{glass}}} &= \frac{v_{\text{water}}}{v_{\text{glass}}} \\ \frac{\sin \theta}{\sin 22} &= \frac{2.27 \times 10^8}{2.1 \times 10^8}; \\ \Rightarrow \theta &= \sin^{-1} \left(\frac{2.27 \times 10^8}{2.1 \times 10^8} \times \sin 22 \right) \\ &= 23.89^\circ \end{aligned}$$

3. A ray of light is incident on a water-glass interface as shown below. Calculate r . (take refractive indices of glass and water $\frac{3}{2}$ and $\frac{4}{3}$ respectively)

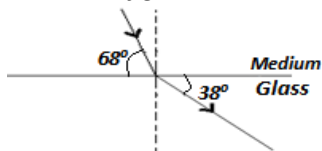


Solution

$$\begin{aligned} n_w \sin \theta_w &= n_g \sin \theta_g \\ \frac{4}{3} \sin 40 &= \frac{3}{2} \sin r \\ r &= \sin^{-1} \left(\frac{\frac{4}{3} \sin 40 \times 2}{3} \right) \\ &= 34.85^\circ \end{aligned}$$

Exercise

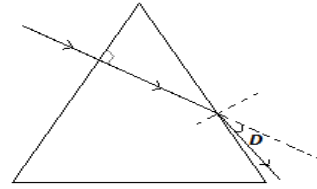
9. If the refractive index of glass is $\frac{3}{2}$, calculate the refractive index of the medium in the figure below



10. Explain why the light bends when it travels from one medium to another.

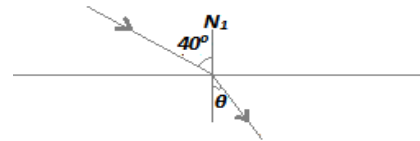
Examples.

with equilateral triangle cross section. Find the angle of deviation D .



6. The speed of light in medium m_1 is $2.0 \times 10^8 \text{ ms}^{-1}$ and the medium m_2 $1.5 \times 10^8 \text{ ms}^{-1}$. Calculate the refractive index of medium m_2 with respect to m_1

7. Calculate angle θ below, given that refractive indices of glass and water are $\frac{3}{2}$ and $\frac{4}{3}$ respectively. Ray is from water to glass

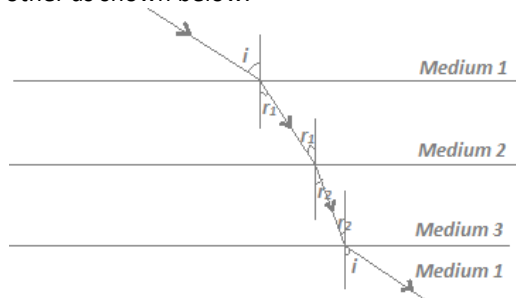


8. A ray of light is incident on a paraffin glass interface as shown in the figure below. Calculate r .



Refraction through successive media

Consider a ray of light travelling through multiple layers of transparent media whose boundaries are parallel to each other as shown below.



$$\begin{aligned} {}_1n_2 &= \frac{\sin i}{\sin r_1}; {}_2n_3 = \frac{\sin r_1}{\sin r_2} \\ {}_1n_2 \times {}_2n_3 &= \frac{\sin i}{\sin r_1} \times \frac{\sin r_1}{\sin r_2} = \frac{\sin i}{\sin r_2} \dots \dots \dots (1) \\ {}_3n_1 &= \frac{\sin r_2}{\sin i}; \Rightarrow {}_1n_3 = \frac{\sin i}{\sin r_2} \dots \dots \dots (2) \end{aligned}$$

Therefore, from equations (1) and (2); ${}_1n_3 = {}_1n_2 \times {}_2n_3$

For k successive media arranged with boundaries parallel; ${}_1n_k$
 $= {}_1n_2 \times {}_2n_3 \dots \dots \dots {}_k-1n_k$

Exercise

1. The refractive index of water is $\frac{4}{3}$ and that of glass $\frac{3}{2}$. Calculate the refractive index of glass with respect to water.

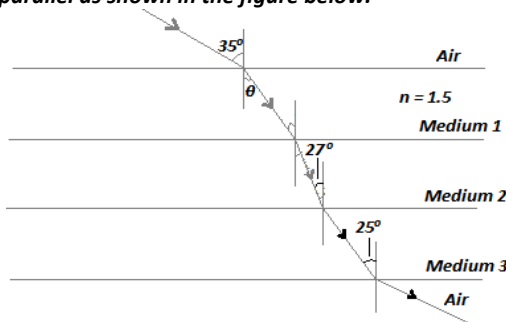
Solution

$${}_w n_g = {}_w n_a \times {}_a n_g$$

$${}_w n_g = \frac{1}{\frac{4}{3}} \times \frac{3}{2};$$

$${}_w n_g = \frac{3}{4} \times \frac{3}{2} = 1.125$$

2. A ray of light travels from air through multiple layers of transparent media 1, 2 and 3 whose boundaries are parallel as shown in the figure below.



Calculate :

- Angle θ
- The refractive index of m_2
- Speed of light in m_1 (speed of light in air = $3.0 \times 10^8 \text{ms}^{-1}$)
- The refractive index of m_3 with respect to m_1

Solutions

$$a) n_a \sin \theta_a = n_1 \sin \theta_1$$

$$1 \sin 35 = 1.5 \sin \theta$$

$$\theta = \sin^{-1} \left(\frac{1 \sin 35}{1.5} \right) = 22.48^\circ$$

$$b) n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$1.5 \sin 22.48 = n_2 \sin 27$$

$$n_2 = \frac{1.5 \sin 22.48}{\sin 27} = 1.263$$

$$c) \frac{\sin \theta_a}{\sin \theta_1} = \frac{v_a}{v_1}$$

$$v_1 = \frac{\sin 35}{\sin 22.48} \times \frac{3.0 \times 10^8}{1} = 2.0 \times 10^8$$

$$d) n_3 = \frac{n_2 \sin \theta_2}{\sin \theta_3};$$

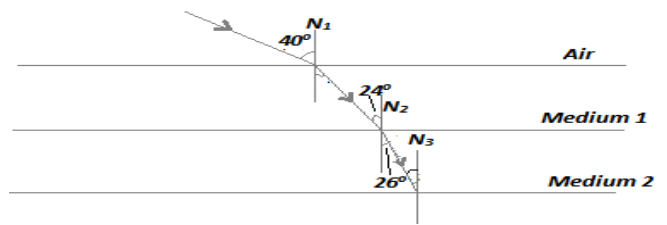
$$\Rightarrow n_3 = \frac{1.263 \sin 27}{\sin 25} = 1.357$$

$${}_1 n_3 = {}_1 n_2 \times {}_2 n_3$$

$${}_1 n_3 = \frac{\sin 22.48}{\sin 27} \times \frac{\sin 27}{\sin 25} = 0.9040$$

Examples.

1. A ray of light travels from air into medium 1 and 2 as shown.

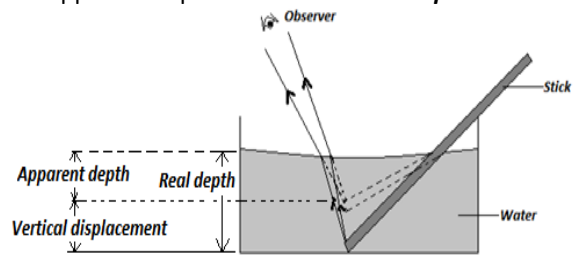


Calculate;

- The refractive index of medium 1
 - Critical angle of medium 1
 - The refractive index of medium 2 relative to medium 1 (${}_1 n_2$)
2. A ray of light from air travels successively through parallel layers of water, oil, glass and then into air again. The refractive indices of water, oil, and glass are $\frac{4}{3}$, $\frac{6}{5}$ and $\frac{3}{2}$ respectively the angle of incidence in air is 60%
- Draw a diagram to show how the ray passes through the multiple layers
 - Calculate:
 - The angle of refraction in water
 - The angle of incidence at the oil glass interface

Refractive Index in Terms of Real and apparent depth

- An object under water or glass block appears to be nearer the surface than it actually is when viewed normally or almost normally. This is due to refraction of light.
- Real depth** is the actual depth of the object in the medium while **apparent depth** is the virtual depth of the object in the medium. The difference between real depth and apparent depth is called **vertical displacement**.



- Refractive indices of materials can be expressed in terms of real and apparent depths.

$$\text{Refractive index, } n = \frac{\text{real depth}}{\text{apparent depth}}$$

Condition for Use of the Formula: This formula applies only when the object is viewed normally.

below

1. A coin in a glass jar filled with water appears to be 24.0cm from the surface of water. Calculate:

I. The height of the water in the jar, given that refractive index of water is $\frac{4}{3}$.

II. Vertical displacement

Solution

I. Refractive index, $n = \frac{\text{real depth}}{\text{apparent depth}}$

$$\frac{4}{3} = \frac{\text{real depth}}{24}$$

$$\text{Real depth} = \frac{4}{3} \times 24 = 32 \text{ cm.}$$

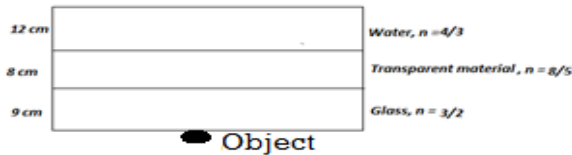
This is the height of water in jar.

II. Vertical displacement

$$= \text{real depth} - \text{apparent depth}$$

$$\text{Vertical displacement} = (32 - 24)\text{cm} = 8 \text{ cm}$$

2. Calculate the displacement and apparent depth of the object shown in the figure below assuming that the object is viewed normally and boundaries of the media are parallel.



Solution

$$\text{apparent depth} = \frac{\text{real depth}}{\text{Refractive index, } n}$$

$$\text{Total apparent depth} = \frac{12}{\frac{4}{3}} + \frac{8}{\frac{3}{5}} + \frac{9}{\frac{3}{2}} = 20 \text{ cm}$$

Vertical displacement

$$= \text{total real depth} - \text{total apparent depth}$$

$$\text{Vertical displacement} = (12 + 8 + 9)\text{cm} - 20 \text{ cm} = 20 \text{ cm}$$

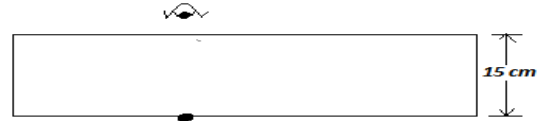
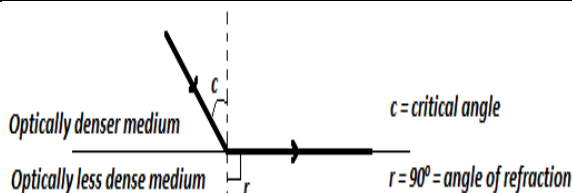
Exercise

1) A tank full of water appears to be 0.5m deep. If the height of water in the tank is 1.0m, calculate the refractive index of water.

2) A glass block of thickened 12cm is placed on a mark drawn on a plain paper. The mark is viewed normally through the glass. Calculate the apparent depth of the mark hence the vertical displacement (refractive index of glass $= \frac{3}{2}$)

3) A beaker placed over a coin contains a block of glass of thickened 12cm. over this block is water of depth 20cm. calculate the vertical displacement of the coin and hence, its apparent depth if it is viewed normally. Assume the boundaries of the media are parallel and take refractive indices of water and glass to be $\frac{4}{3}$ and $\frac{3}{2}$ respectively

4) A mark on a paper is viewed normally through a rectangular block of a transparent material as shown



If the speed of light in the material is $1.25 \times 10^8 \text{ ms}^{-1}$ calculate:

- a) The apparent depth of the mark
- b) The vertical displacement of the mark (speed of light in air $= 3.0 \times 10^8 \text{ ms}^{-1}$)

5) A pin is placed at the bottom of a tall parallel sided glass jar containing a transparent liquid when viewed normally from the top, the pin appears nearer the surface than it actually is: With the aid of diagram, explain this observation

6) The table below shows the results obtained when such an experiment was carried out using various depths of the liquid

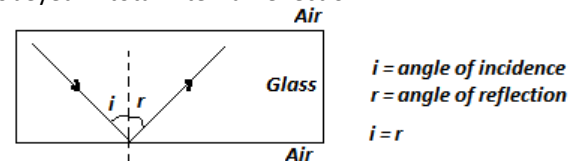
Real depth (cm)	4.0	6.0	8.0	10.2	12.8
Apparent depth (cm)	2.44	3.66	4.88	6.10	7.32

- a) Plot a graph of apparent depth against real depth
- b) Using the graph, determine the refractive index of the liquid
- c) What is the vertical displacement of the pin when the apparent depth is 1.22cm?

7) How long does it take a pulse of light to pass through a glass block 15cm in length?(Refractive index of glass is 1.5 and velocity of light in air is $3.0 \times 10^8 \text{ ms}^{-1}$)

Total Internal Reflection

- Total internal reflection refers to the complete bouncing off of light at the boundary between two media in the optically denser medium. The laws of reflection are obeyed in total internal reflection.



Conditions for Total Internal Reflection

- For internal reflection to occur:
 - a) Light must be travelling from optically denser to optically less dense medium.
 - b) The angle of incidence in the optically denser medium must be greater than the critical angle

Critical Angle

Critical angle is the angle of incidence in optically denser medium for which the angle of refraction in the optically less dense medium is 90°

Solution

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

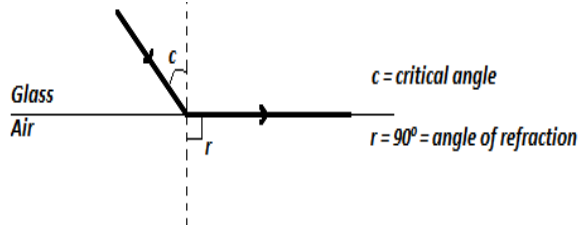
$$2.4 \sin c = 1.467 \sin 90$$

$$c = \sin^{-1} \left(\frac{1.467 \sin 90}{2.4} \right) = 37.66^\circ$$

Exercise

Relationship between Critical Angle and Refractive Index

- Consider a ray of light striking a glass-air interface as shown below



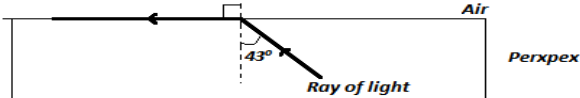
From Snell's law,

- $n_g n_a = \frac{\sin c}{\sin 90^\circ}$
- But $n_a n_g = 1 / n_g n_a = \frac{\sin 90^\circ}{\sin c}$
- $\sin 90^\circ = 1$
- $n_a n_g = \frac{1}{\sin c}$

Thus, refractive index, $n = \frac{1}{\sin C}$

Examples

- The figure below shows the path of a ray of light passing through a rectangular block of Perspex placed in air



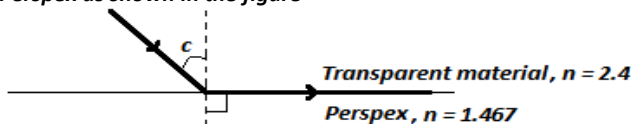
Calculate the refractive index of Perspex

Solution

$$n = \frac{1}{\sin c}$$

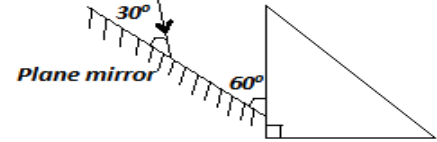
$$n = \frac{1}{\sin 43} = 1.467$$

- A ray of light travels from a transparent medium into Perspex as shown in the figure



- Which of the two media is optically denser?
Transparent material
- Calculate the critical angle C

- What do you understand by the term total internal reflection?
- State the conditions necessary for total internal reflection
- Define critical angle. Derive an expression for the relationship between critical angle and refractive index
- The figure below shows a plane mirror at 30° to face of a right angled isosceles prism of refractive index 1.50. Complete the path of light ray after reflection.

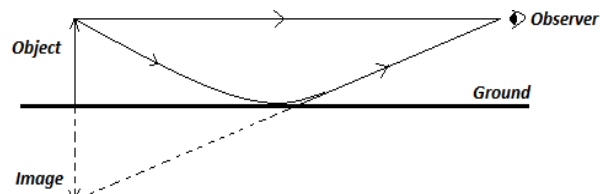


- Calculate critical angle for diamond-water interface ($n_w = 1.33$, $n_d = 2.46$)

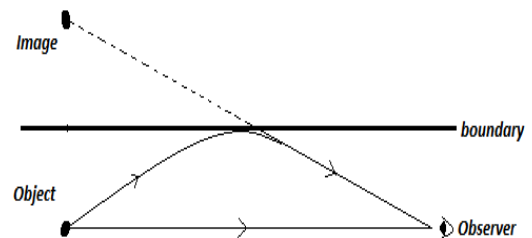
Effects of Total Internal Reflection

1. Mirage

- Mirage refers to optical illusion of an inverted pool of water that is caused by total internal reflection light.
- During a hot day air near the ground is warmer and therefore physically less dense than air away from the ground. Therefore on a hot day the refractive index increases gradually from the ground upwards.
- A ray of light travelling in air from sky to ground undergoes continuous refraction and finally reflected internally.

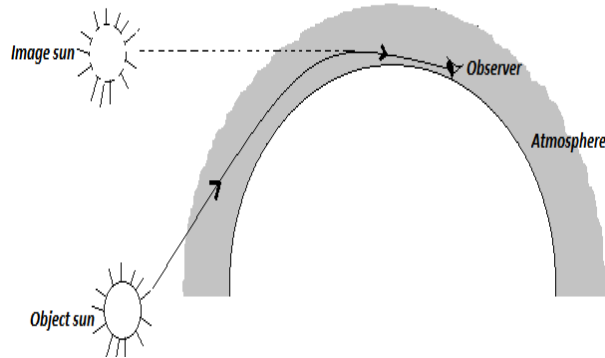


- Mirages are also witnessed in very cold regions in which the refractive index increases gradually from the ground upwards. Images appear inverted in the sky.



2. Atmospheric Refraction

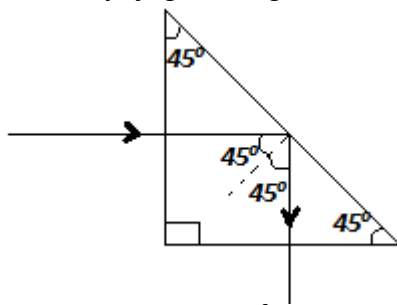
- This is a phenomenon in which light rays from the sun are refracted and then reflected internally towards the earth. As a result, the sun is seen even after it has set or before it rises.



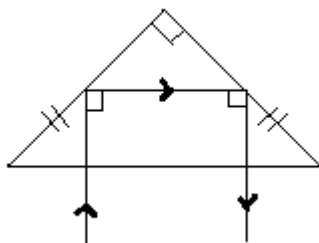
Total Internal Reflection Prisms

- Right-angled isosceles glass or Perspex prisms are used.

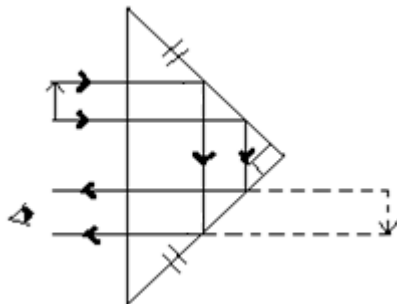
I. To turn a ray of light through 90°



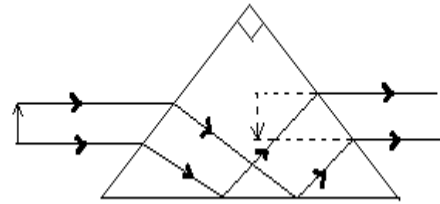
II. To turn a ray through 180°



III. Inversion with deviation



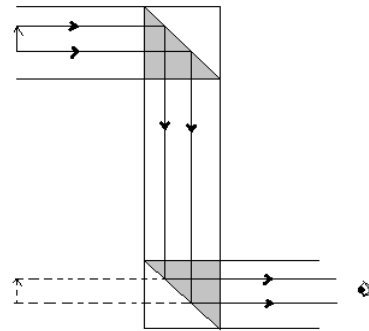
IV. Inversion without deviation



Applications of Total Internal Reflection

1. Periscope

- Light is deviated through 90° by first prism before the second prism deviates it further through 90° in the opposite direction. An upright virtual image is formed

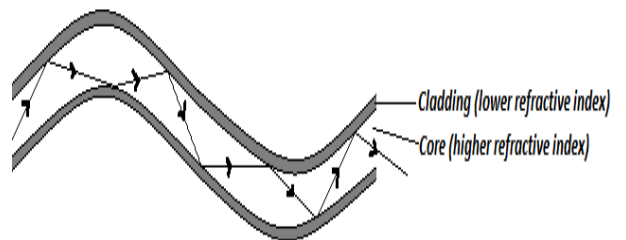


Note: Prisms are preferred to plane mirrors for use in periscopes and other optical instrument because:

- Mirrors absorb some of the incident light
- The silvering on mirrors can become tarnished and peel off
- Thick mirrors produce multiple images

2. Optical Fibre

- An optical fibre is a thin flexible glass rod of small diameter in the order of $10^{-6}m$. The central case of the glass is coated with glass of lower refractive index (**cladding**)



- A ray of light entering the fibre undergoes total internal reflections on the boundary of the high and low refractive index glass. The light therefore travels through the entire length of the fibre without any getting lost.

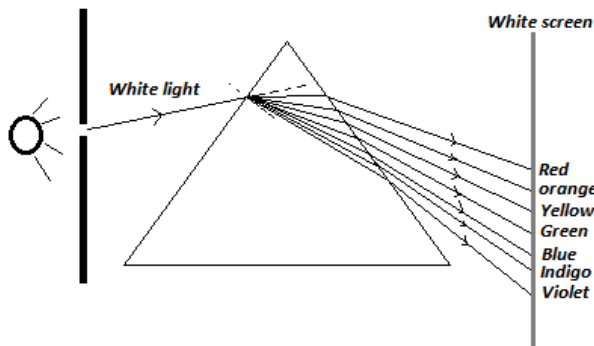
Advantages of optic fibers over ordinary cables.

- they have high carrying capacity
- they are thinner and lighter

- I. Used in medicine to view internal organs of the body.
- II. Used in telecommunication where they have higher advantage than ordinary cables since they have higher carrying capacity, they are thinner and lighter.

Dispersion of White Light

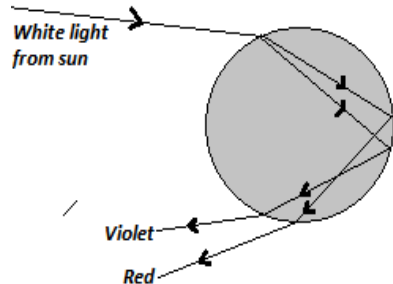
- Dispersion of light is the splitting of white light into its component colors. .White light is a mixture of seven colors.
- The components of white light travel with same velocity in vacuum but their velocities are not the same in other media.



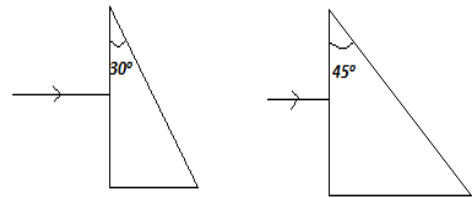
- **Cause of Dispersion of White Light:** The separation of white light into constituent colours is due to their different velocities in a given transparent medium.
- The **velocity of red is highest** while that of **violet is the least**. **Red colour has longest wavelength** while **violet the least wavelength** ($v = \lambda f$)

The Rainbow

- Rainbow is a bow-shaped colour band of the visible spectrum seen in the sky. It is formed when white light from the sun is **refracted, dispersed and totally internally reflected** by rain drops.



1. The diagram below show two prisms



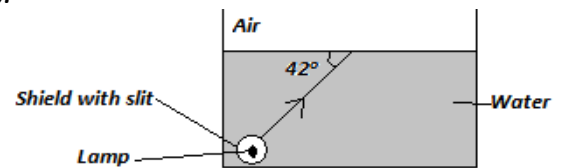
Given that the critical angle of the glass in both prisms is 42° sketch the paths of the two beams of monochromatic light until they leave the flasks.

2. The figure below show how white light behaves when it is incident on a glass prism.



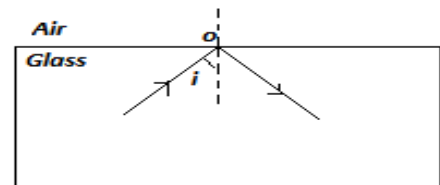
- I. Determine the critical angle of the glass material
- II. Determine the refractive index of the glass material

3. The diagram below shows a transparent water tank containing water. An electric damp covered with a shield which has a narrow slit fixed at one near of the tank. A light ray from the slit reaches the water surface at an angle of 42° as shown below.



- I. Determine the angle of refraction for the ray shown in the diagram
- II. Determine the angle of incidence for which the angle of refraction is 90° (refractive index of water =1.33)

4. The diagram below shows a ray of light incident on the glass – air interface from the inside of the glass. The angle of incidence i , is slightly smaller than the critical angle of glass.



State and explain what would be observed on the ray if a drop of water was placed at the point of incidence, o