MOTION

Motion refers to the movement of bodies.
It is change of a body's position from one place to another.

TERMS USED:

DISTANCE.

This is the length between 2 fixed points.
Distance is a scalar quantity and measured in metres.

DISPLACEMENT.

This is the distance covered in a specific direction
Displacement is a vector quantity and measured in metres.

SPEED.

This is the rate of change of distance with time.

$$\text{speed} = \frac{\text{distance}}{\text{time taken}}$$

Speed is a scalar quantity and the SI unit of speed is metres per second (ms^{-1})

Note:

1. If the speed of an object changes, we use the average speed. Average speed is the ratio of total distance moved by a body over the time it takes to cover that distance.
   Taking V as the final speed and U as the initial speed, then we have

   $$\text{Average speed } = \frac{\text{total distance covered}}{\text{total time taken}}$$

   OR Average speed $= \frac{V+U}{2}$

2. Uniform speed. This is when the rate of change of distance with time is constant.
   OR: This is when the rate at which an object covers equal distances in equal time intervals is constant

Examples

1. A car covers 72km in 2hours. Find its speed in m/s.

   Solution

   $$\text{speed} = \frac{\text{distance}}{\text{time taken}} = \frac{72 \text{ x } 1000}{2 \text{ x } 60 \text{ x } 60} = 10ms^{-1}$$

2. A car increase its speed steadily from 30 km/hr to 60km/hr in one minute
   (a) Determine its average speed during this time in
   (i) km/hr
   (ii) mls
   (b) how far does it travel whilst increasing its speed

   Solution

   u = 30km/hr, v = 60 km/hr

   (a) (i) average speed = $\frac{v + u}{2} = \frac{60+30}{2}$ = 45km/ hr

(ii) Average speed = $\frac{45 \text{ x } 1000 \text{ m}}{1 \text{ x } 60 \text{ x } 60 \text{ s}}$ = 12.5m/s

(b) Distance = speed x time = 12.5 x 60 = 750m

3. A girl runs 120m in a quarter of a minute, she then rests for 20s and completes the remaining 80m in another quarter a minute. Find her average speed

Solution

Total distance = 120 + 80 = 200m

Total time taken = $\left[\left(\frac{1}{4} \text{ x } 60\right) + 20 + \left(\frac{1}{4} \text{ x } 60\right)\right] = 15 + 20 + 15 = 50\text{s}$

$$\text{Average speed} = \frac{\text{total distance covered}}{\text{total time taken}} = \frac{200}{50} = 4 \text{ ms}^{-1}$$

VELOCITY:

Velocity is the rate of change of distance with time in a specified or given direction.
OR
Velocity is the rate of change of displacement with time.

$$\text{velocity} = \frac{\text{displacement}}{\text{time taken}}$$

Velocity is a vector quantity and the SI unit of velocity is metres per second (ms^{-1})

Types of velocities

(i) Initial velocity (U)
Initial velocity is the velocity of a body at the beginning of a journey.
OR it is the rate of change of displacement with time at the beginning of a journey.

(ii) Final velocity (V)
This is the velocity of a body at the end of a journey
Or It is the rate of change of displacement with time at the end of a journey.
Change in velocity = V − U

Example
A body starts moving with a speed of $2ms^{-1}$ and after 4 seconds it attained a velocity of $5ms^{-1}$. Calculate the change in velocity.
Solution
Change in velocity = V – U = 5 – 2 = $3ms^{-1}$

(iii) uniform velocity
This is the rate of change of displacement with time, which is constant.
Or
Uniform velocity is the constant rate of change of displacement with time.

(iv) Non uniform velocity
This is the rate of change of displacement with time which is not constant.

(v) Average velocity

This is the ratio of total displacement to total time taken to cover that distance.

$$\text{Average speed} = \frac{\text{total distance covered}}{\text{total time taken}}$$

$$\text{OR Average speed} = \frac{V + U}{2}$$, where; $V = \text{final velocity}$, $U = \text{initial velocity}$.

Note;

- For a body starting from rest the initial velocity "u' must be zero i.e. u = 0 ms^{-1}
- For a stationary body, starting motion means that the body is starting from rest u = 0 ms^{-1}
- For a body traveling with a certain velocity, x, the initial velocity for such a body will be x so, u = x ms^{-1} e.g. a car traveling at 20 ms^{-1}, has u =20 ms^{-1}
- If a body comes or is brought to rest, its final velocity v = 0 ms^{-1}

ACCELERATION (a)

It is the rate of change of velocity with time.

$$\text{Acceleration a} = \frac{\text{change in velocity}}{\text{time taken}}$$

$$\text{Acceleration a} = \frac{\text{final velocity} - \text{initial velocity}}{\text{time taken}}$$

$$\text{Acceleration } a = \frac{v - u}{t}$$

The SI unit of acceleration is metres per seconds squared (ms^{-2})

NB: When the velocity of a body is changing, the body is said to be accelerating.

Acceleration is: *Positive* *- if the velocity is increasing and*
Negative *- if the velocity is decreasing.*

A negative acceleration is called deceleration or retardation.

Uniform Acceleration

This is when the rate of change of velocity with time is constant.

Or

It is the acceleration of a body whose velocity increases or decreases by a constant value per second.

Non – uniform acceleration

This is when the rate of change of velocity with time is not constant.

Or

It is acceleration of a body when its velocity does not increase or decrease by a constant value.

Examples

1. A train has a speed of 72km/h and its speed after 4s is 144km/h. Find its acceleration.

Solution

Final velocity, v = 144km/h = $\frac{144 \ x\ 1000}{1\ x\ 3600}$ = 40m/s

Initial velocity, u = 72km/h = $\frac{72\ x\ 1000}{1\ x\ 3600}$ = 20m/s

Acceleration $a = \frac{v-u}{t} = \frac{40-20}{4} = 5ms^{-2}$

2. A car from rest moves to a velocity of 20m/s in 4s, find its acceleration.

Solution

Final velocity, v = 20m/s

Initial velocity, u = 0m/s

Acceleration $a = \frac{v-u}{t} = \frac{20-0}{4} = 5ms^{-2}$

3. A motor car is uniformly retarded and brought to rest from a speed of 108km/hr in 15 seconds. Find its acceleration.

Solution

Initial velocity, U = $\frac{108\ x\ 1000}{1\ x\ 60\ x\ 60} = \frac{1080}{36}$ = 30m/s

Final velocity, V = 0m/s

Acceleration = $\frac{v-u}{t} = \frac{0-30}{15} = -2ms^{-2}$

The minus (-) sign means the car is accelerating in opposite direction to its initial velocity i.e the body is decelerating.

Exercise:

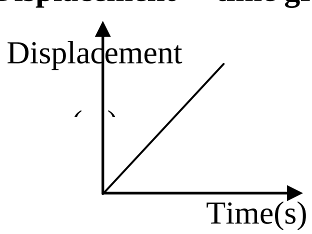
1. A car is uniformly accelerated from rest and acquires a velocity of 50ms^{-1} after 10s. Calculate his acceleration
2. A cyclist riding at 10ms^{-1} accelerates to 90ms^{-1} in a quarter a minute. Calculate his acceleration
3. A bus travelling at 360kmh^{-1} is brought to rest in 1minute, calculate the deceleration or retardation

GRAPHICAL REPRESENTATION OF LINEAR MOTION

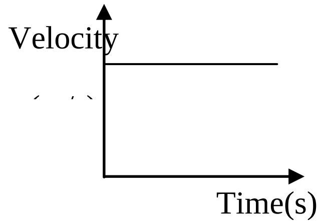
Graphs can be used to represent the motion of a body which is moving in a straight line.

1. UNIFORM VELOCITY

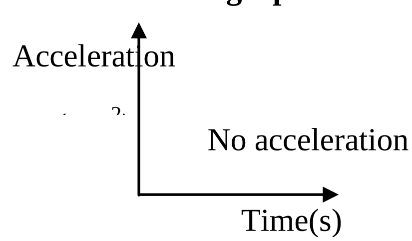
Displacement – time graph



Velocity – time graph



Acceleration – time graph

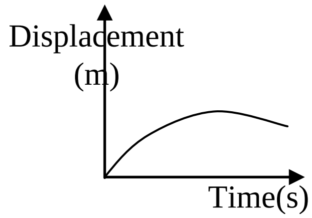


Note:

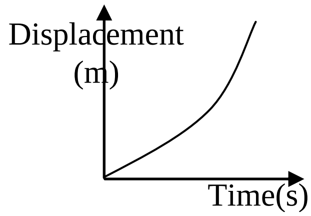
Uniform velocity means that acceleration is zero

The slope of a distance – time graph gives the velocity of the body

2. NON – UNIFORM VELOCITY



OR



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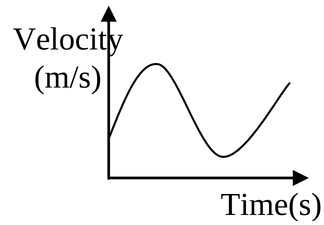
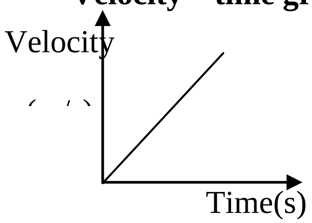
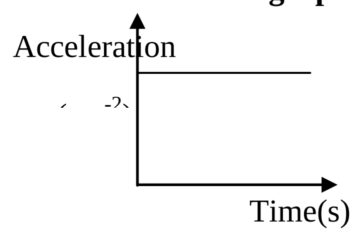


3. UNIFORM ACCELERATION AND UNIFORM DECELERATION

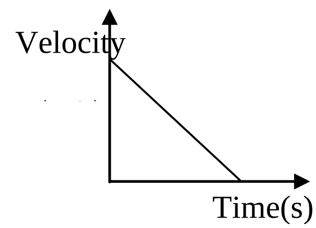
Velocity – time graph



Acceleration – time graph



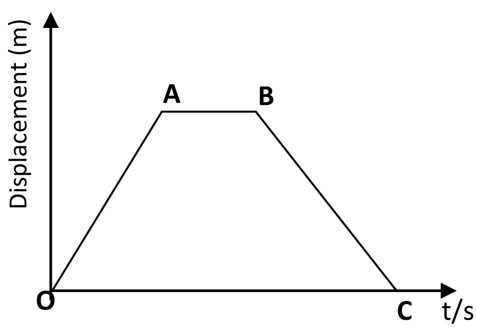
Constant deceleration



Note:

- Uniform acceleration implies that the velocity is increasing or its non – uniform
- The slope of a velocity – time graph gives the acceleration

Displacement / distance – time graphs can be combined to form a single graph with the different stages of the motion as shown below.

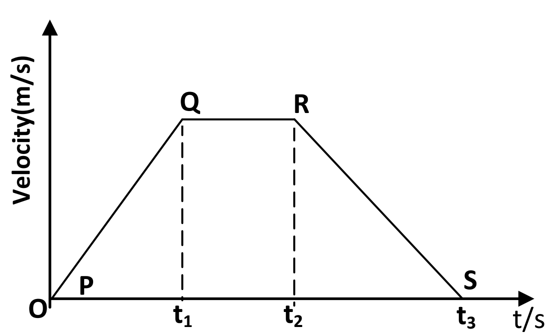


OA - The slope is positive,
- The body is moving with constant velocity during this period.

AB - The slope is zero i.e the velocity is zero
- The body is stationary

BC - The slope is negative
- The body is moving with constant velocity in the opposite direction during this period

The graphs for uniform acceleration, velocity and deceleration can be combined to represent a linear motion for a body starting from rest or at a certain initial velocity.
The total distance travelled is equal to the area under the graph and can be calculated using appropriate formulae or using equations of motion.



Stage	State of the body	Interpretation
P - Q	The body starts from rest and moves with a uniform acceleration during this period.	- The velocity is positive and increasing. - The slope is positive and constant. - The acceleration is positive and constant.
Q - R	The body is moving with a constant velocity during this period.	- The velocity is positive and constant. - The slope is zero. - The acceleration is zero.
R - S	At t_2, the body brakes and is uniformly decelerated to rest at t_3.	- The velocity is positive and decreasing. - The slope is negative and constant. - The acceleration is negative and constant. - The negative acceleration means that it acts in the opposite direction the direction of velocity.

Examples

1. A car starts from rest and steadily accelerates for10s to a velocity of 20m/s. It continues with this velocity for a further 20s before .it is brought to rest in 20s.

(a) Draw a velocity time graph to represent this motion.

(b) Calculate

(i) Acceleration

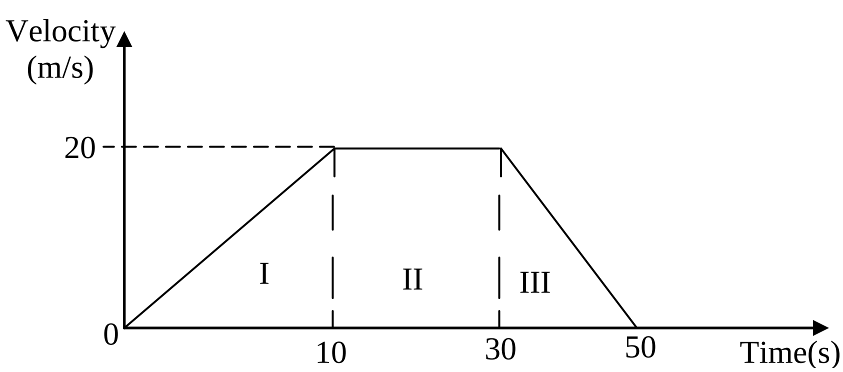
(ii) Deceleration

(iii) Distance travelled

(iv) Average speed.

Solution

(a)



(b) (i) Acceleration, $a = \frac{v - u}{t} = \frac{20 - 0}{10} = \frac{20}{10} = 2ms^{-2}$

(ii) Deceleration, $a = \frac{v - u}{t} = \frac{0 - 20}{10} = \frac{-20}{20} = -1ms^{-2}$

(iii) Distance traveled, s = area under a velocity – time graph

= Area of (I + II + III)

S = ½bh + l w + ½bh

= (½ x 10 x 20) + (20 x 20) + (½ x 20 x 20)

= 100 + 400 +200

= 700m

(iv) Average speed = $\frac{\text{total distance}}{\text{total time}} = \frac{700}{50} = 14\text{ms}^{-1}$

2. A car from rest accelerates to velocity 30m/s in 10s. It continues at uniform velocity for 30s and then decelerate so that it stops in 20s

(a) Draw a velocity time graph to represent its motion

(b) Calculate

(i) Acceleration

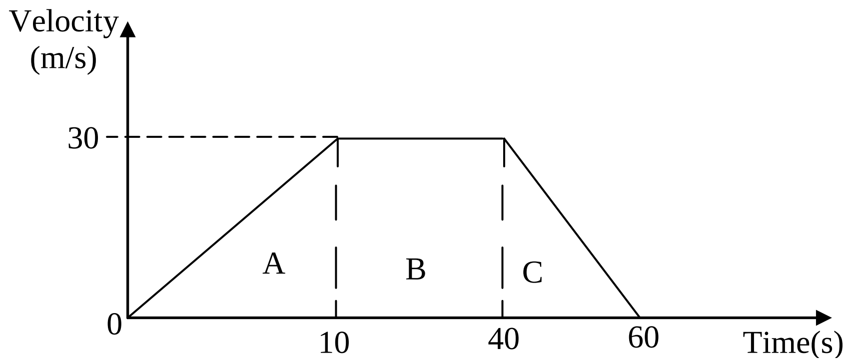
(ii) Deceleration

(iii) Distance travelled

(iv) Average speed.

Solution

(a)



(b) (i) Acceleration, $a = \frac{v-u}{t} = \frac{30-0}{10} = 3\text{ms}^{-2}$

(ii) Deceleration, $a = \frac{\text{v}-\text{u}}{\text{t}} = \frac{0-30}{20} = \frac{-30}{20} = -1.5\text{ms}^{-2}$

(iii) Distance travelled = total area under graph

= Area of (A + B + C)

= ½bh + *l* w + ½bh

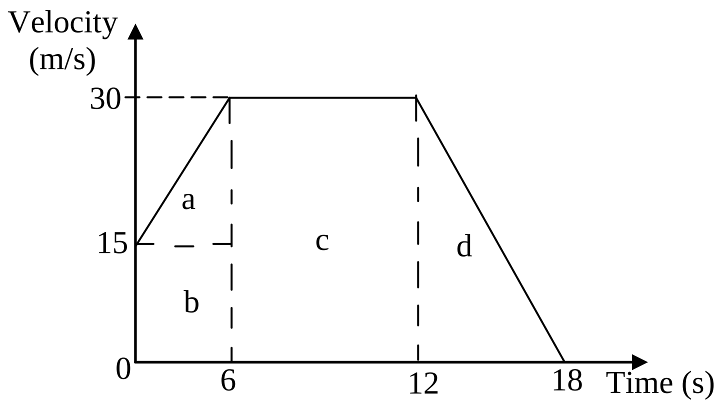
= (½ x10 x30) + (30 x30) + (½x20 x30)

=150 + 900 + 300

= 1350m

(v) Average speed = $\frac{\textit{total distance}}{\textit{total time}} = \frac{1350}{60} = 22.5\text{ms}^{-1}$

3. The graph below represents a velocity time graph of a body in motion.



(i) Describe the motion of the body.
(ii) Calculate the total distance travelled.
(iii) Determine the average speed.

Solution

(i) The body started moving with an initial velocity of 15 m/s and uniformly accelerated to a velocity of $30ms^{-1}$ in 6s. The body then moved with a constant velocity of $30ms^{-1}$ for 6s. It then decelerated uniformly to rest in 6s.

(ii) Total distance travelled = total area under graph

= Area of (a + b + c + d)

$= \frac{1}{2} bh + lw + l\,w + \frac{1}{2}bh$

=(½ x 6 x 15) + (15 x 6) + (30 x 6) + (½ x 6 x 30)

= 45 +90 + 180 + 90

=135 + 270

= 405m

(iii) Average speed = $\frac{total\ distance}{total\ time} = \frac{405}{18} = 22.5ms^{-1}$

Graphical calculations

1. A body starts from rest and accelerates uniformly for 5s when its velocity is $20ms^{-1}$. It travels at this speed for 15s and is then brought to rest by uniform retardation in 10s.

(a) Sketch its velocity-time graph

(b) From the graph, find;

(i) Acceleration
(ii) Retardation
(iii) Total distance covered
(iv) Average speed

2. A car runs at a constant speed for $15ms^{-1}$ for 300s, it then accelerates uniformly to a velocity of $25ms^{-1}$ in a period of 20s. This speed is maintained for 300s before the car is brought to rest in 50s.

(a) Sketch its velocity-time graph

(b) From the graph, find;

(i) Acceleration
(ii) Retardation
(iii) Total distance covered
(iv) Average speed

3. A body moves with a constant velocity of $2ms^{-1}$ for 10s and then rests for 10s and later moves with a constant velocity of $1ms^{-1}$ for 10s in the opposite direction.

(a) Sketch the velocity-time graph

(b) Find the;

(i) Find the average speed
(ii) Find the displacement

4. A car accelerates uniformly from rest with an acceleration of $1.5ms^{-1}$ for 20s, it then travels at a constant velocity for 2minutes before slowing down with uniform deceleration to come to rest in further 10s
   (a) Sketch the velocity-time graph
   (b) Find the;
      (i) Constant velocity (speed)
      (ii) Total distance and average speed

5. Two vehicles **X** and **B** accelerate uniformly from rest. Vehicle **A** attains a maximum velocity of 30m/s in 10s while **B** attains a maximum velocity of 40m/s in the same time. Both vehicles maintained these velocities for 6s. They are then decelerated such that **X** comes to rest after 6s while **B** comes to rest after 4s.
   (a) Sketch on the same axes a velocity-time graph for the motion of the vehicles.
   (b) Calculate the velocity of each vehicle 18s after the start.
   (c) How far will the 2 vehicles be from one another during this moment in (ii) above?

EQUATIONS OF MOTION

Suppose an object with a velocity **u** moves with uniform acceleration **a** for a time **t** and attains a velocity **v**.

1st equation

From the definition of acceleration,

$$\text{Acceleration } \boldsymbol{a} = \frac{\text{change in velocity}}{\text{time taken}}$$

$$\text{Acceleration } a = \frac{v-u}{t}$$

$$\Rightarrow at = v - u$$

$$\therefore \boldsymbol{v} = \boldsymbol{u} + \boldsymbol{at} \text{ ....(i)}$$

2nd equation

The distance **s** travelled by the object is given by,

$$\text{Displacement} = \text{average speed x time}$$

$$\text{average velocity} = \frac{v+u}{2}$$

$$\Rightarrow s = \left(\frac{v+u}{2}\right)t$$

$$\text{But } v = u + at$$

$$\Rightarrow s = \left(\frac{u + at + u}{2}\right)t$$

$$s = \frac{(2u+at)t}{2}$$

$$s = \frac{2ut + at^2}{2}$$

$$\therefore \mathbf{s} = \mathbf{ut} + \frac{\mathbf{1}}{\mathbf{2}}\mathbf{at^2} \text{ .... } (\mathbf{ii})$$

3rd equation

From $s = \left(\frac{v+u}{2}\right)t$,

Since $v = u + at$,

$$t = \frac{v-u}{a}$$

$$\Rightarrow s = \left(\frac{v+u}{2}\right) \text{ x } \left(\frac{v-u}{a}\right)$$

$$s = \frac{(v+u)(v-u)}{2a}$$

$$s = \frac{v^2 - u^2}{2a}$$

$$2as = v^2 - u^2$$

$$\therefore \mathbf{v^2} = \mathbf{u^2} + \mathbf{2as} \text{ .... } (\boldsymbol{iii})$$

Worked Examples

1. An object starts from rest and moves with an acceleration of 10 m/s^2 for four seconds.

Calculate: (a) The velocity

(b) The distance traveled

Solution:

$u = 0m/s$, $a = 10m/s^2$, $t = 4s$

(a) Using the formula $v = u + at$,

$\Rightarrow v = 0 + (10 \text{ x } 4)$

$\therefore$ The velocity $v = 40m/s$

(b) Using the formula $s = ut + \frac{1}{2}at^2$

$s = 0 \text{ x } 4 + \frac{1}{2} \text{ x } 10 \text{ x } 4 \text{ x } 4 \quad = 80m$

2. A car moving with uniform acceleration of 4 m/s^2 increases its velocity from 20 m/s to 60 m/s, Calculate:

(a) the total time taken during this change

(b) the total distance moved

Solution:

$a = 4 \ m/s^2$, $u = 20$ m/s, $v = 60$ m/s, $t = ?$, $s = ?$

(a) Using the formula $v = u + at$

$60 = 20 + 4 \text{ x } t$

$60 = 20 + 4t$

$4t = 60 - 20$

$4t = 40$

$\therefore$ **t = 10 s**

(b) Using the formula $s = ut + ½at^2$

$= 20 \text{ x } 10 + ½ \text{ x } 4 \text{ x } 10 \text{ x } 10$

$= 200 + 200$

$\therefore$ **s = 400 m**

3. A body traveling with a velocity of 10m/s is uniformly retarded to rest in 5 seconds Find;

(a) its acceleration.

(b) the distance traveled during the retardation.

Solution:

$u = 10$ m/s, $v = 0$, $t = 5$ s, $a =?$, $s = ?$

(a) Using the formula $v = u + at$

$0 = 10 + 5$

$\frac{5a}{5} = \frac{-10}{5}$

$\therefore$ **a = -2 m/s^2**

The minus sign means that the body is accelerating in the opposite direction to its initial velocity.

(b) Using the formula: $v^2 = u^2 + 2as$

$0 = 10^2 + 2 \text{ x } 2 \text{ x } s$

$0 = 100 + 4s$

$4s = 100$

$\frac{4s}{4} = \frac{100}{4}$

$\therefore$ **s = 25 m**

4. A racing car starts from rest and moves with uniform acceleration of $3ms^{-2}$ for 4 seconds. Then moves with uniform velocity for 2 second. It is brought to rest after a further 2 seconds

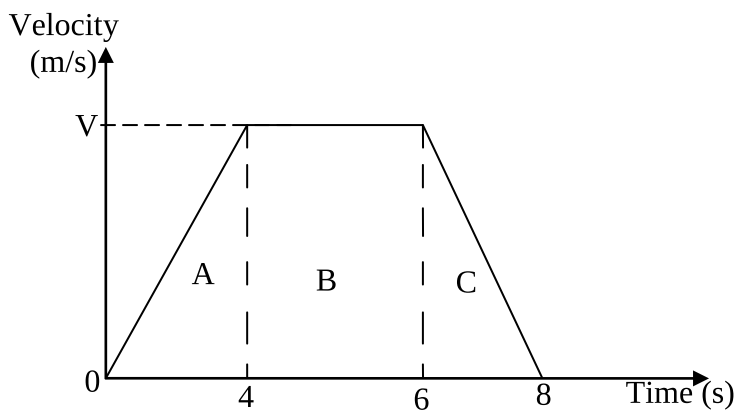
(a) Draw a velocity time graph for motion of the car.

(b) Find total distance travelled.

(c) Average speed.

Solution

(a)



(b) From V = u + at

= 0 + 3 x 4

= 12m/s

Total distance covered = total area under a velocity time graph.

= Area of (A + B + C)

= ½bh + lw + ½bh

= ½ x 4 x 12 + 12 x 2 + ½ 12 x 2

= 24 + 24 + 12

= 60m

(c) Average speed $= \frac{tatal\ distance}{total\ time} = \frac{15}{8} = 1.875ms^{-1}$

5. A body of mass 60 kg starts moving with an initial velocity of 15 m/s and accelerates at a rate of $4m/s^{-2}$ in 5s. It then maintains a constant velocity for another 5s and brought to rest in 7s.

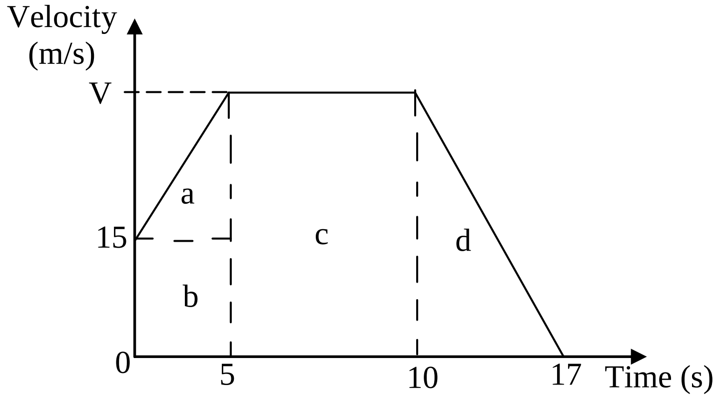
(i) Draw a velocity –time graph to represent this motion.

(ii) Calculate the total distance travelled

(iii) Calculate the retarding force

Solution

(i)



(ii) From V = u + at

$\Rightarrow V = 15 + 4\ x\ 5 = 35ms^{-1}$

Total distance = total area under the graph.

= Area of (a + b + c + d)

= *½ bh + lw +l w + ½bh*

= (½ x 5 x 20) + (15 x 5) + (35 x 5) + (½ x 7 x 35)

= 50 + 75 + 175 + 122.5

= 422.5m

(iii) Deceleration, $a = \frac{v-u}{t} = \frac{0-35}{7} = \frac{-35}{7} = -5ms^{-2}$

Retarding force, F = ma = 60 x 5 = 300N

Exercise

1. A girl started running from point **A** with a velocity of 40m/s with a uniform deceleration of $2m/s^2$. She passed point **B** after 10s. Calculate the girl's velocity at **B**.

2. A body started from rest and moved with a uniform acceleration of $2m/s^2$.
(a) What is its velocity after 5s?
(b) How far has it travelled in this time?
(c) After how long will the body be *100m* from its starting point.

3. A car started from 5 m/s with an acceleration of $3m/s^2$ for 2s.
(a) What is its velocity after 2s?
(b) How far has it travelled in this time?

4. A body moving with a velocity of $8ms^{-1}$ has an acceleration of $3.2ms^{-1}$, find the
(a) Distance covered in the 4^{th} second
(b) Distance covered between the 4^{th} and 9^{th} second

5. A cyclist of mass 60kg starts from rest and accelerates at $5ms^{-2}$ for 20s, calculate the;
(a) Final velocity
(b) Distance travelled
(c) Kinetic energy gained
(d) Work done

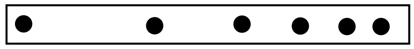
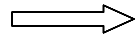
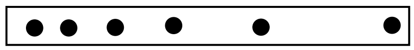
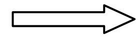
6. A car is travelling at $90kmh^{-1}$ when the brakes are applied, it retards at $2ms^{-1}$
(a) How long does it take for the car to stop (come to halt)
(b) How far does it travel from the instant the brakes are applied

TICKER-TAPE TIMER (TICKER – TAPE VIBRATOR)

This is a device used to investigate speed, velocity and acceleration of a moving body.

It consists of a steel strip which vibrates at mains frequency, *f*, making dots on paper tape which is pulled by the body.

Motion of ticker tape

State of motion	Sample tape	Direction of motion
Uniform velocity		
Uniform acceleration		
Uniform deceleration		

Terms used:

- **Frequency (f):** These are vibrations per second.
  It is also the number of dots printed per second. The S I unit is Hertz (Hz).

Example: A frequency of 60Hz means 60 dots per second.

- **Tick /period (T):** This is the time interval between any two successive dots. The SI unit of period is seconds.

  Tick, $T = \frac{1}{\text{frequency,f}}$

  $\Leftrightarrow\ T = \frac{1}{f}$

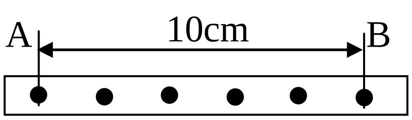
NB: The time taken to make the dots from the formula

Total time taken = number of spaces between reference points x one tick(period)

$\Leftrightarrow t = nT$

Examples:

1. The ticker tape shown in the figure was pulled through a ticker timer, which makes 50 dots per second. Calculate the speed at which the tape is pulled.



Solution

Frequency, f = 50Hz,

Period, $T = \frac{1}{f} = \frac{1}{50} = 0.02\text{s}$

Time taken, t = number of spaces between reference points x period

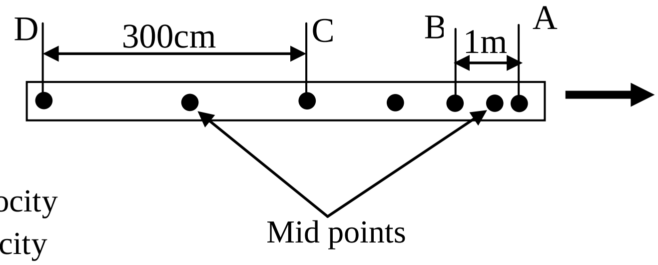
= 5 x 0.02

= 0.1s

Total distance covered = 50 cm $= \frac{50}{100}$ m = ***0.5 m***

$\text{Speed} = \frac{\text{distance covered}}{\text{time taken}} = \frac{0.5}{0.1} = 5\text{ms}^{-1}$

2. Below is a tape printed by a ticker timer vibrating at 20Hz.



Calculate the;

(i) Initial velocity

(ii) Final velocity

(iii) Acceleration

Solution

Frequency, f = 20Hz,

Period, $T = \frac{1}{f} = \frac{1}{20} = 0.05s$

(i) Let $d_1 = 1m$,

Number of spaces $n_1 = 2$

Time taken, $t_1 = n_1T = 2 \times 0.05 = 0.1s$

Initial velocity, $u = \frac{d_1}{t_1} = \frac{1}{0.1} = 10ms^{-1}$

(ii) Let $d_2 = 300cm = \frac{300}{100}m = 3m$,

Number of spaces $n_2 = 2$

Time taken, $t_2 = n_2T = 2 \times 0.05 = 0.1s$

Final velocity, $v = \frac{d_2}{t_2} = \frac{3}{0.1} = 30ms^{-1}$

(iii) $$\begin{matrix} Time\ taken \\ for\ change \end{matrix} = \begin{pmatrix} number\ of\ spaces \\ between\ mid\ points\ of \\ AB\ and\ CD \end{pmatrix} x\ period$$

$\Leftrightarrow t = nT = 4 \times 0.05$

$t = 0.2s$

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken for change}}$$

$$a = \frac{v - u}{t} = \frac{30 - 10}{0.2} = \frac{20}{0.2}$$

Thus, acceleration, $a = 100ms^{-2}$

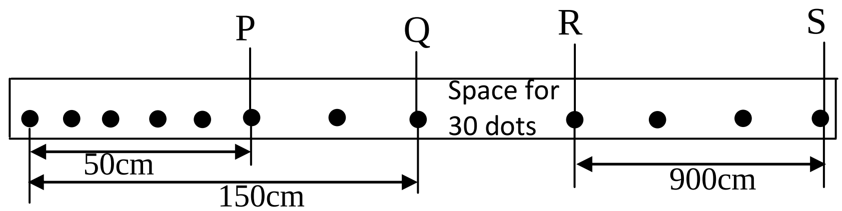
NB:

- If there are n-dots, then there are (n – 1) spaces. i.e, $_s = (\boldsymbol{n_d} - \mathbf{1})$.

  Where $\mathbf{n_s}$ is the number of spaces and $\mathbf{n_d}$ is the number of dots.

- Usually, the first and last section of the tape are ignored in experiments because the motion of the trolley is unsteady and the dots are near each other.

3. The timer is vibrating at 20Hz. Calculate the acceleration.



Solution

Frequency, f = 20Hz,

Period, $T = \frac{1}{f} = \frac{1}{20} = 0.05s$

Let $d_1 = 150 - 50 = 100cm$,

$d_1 = \frac{100}{100} = 1m$,

Number of spaces $n_1 = 2$

Time taken, $t_1 = n_1T = 2 \times 0.05 = 0.1s$

Initial velocity, $u = \frac{d_1}{t_1} = \frac{1}{0.1} = 10ms^{-1}$

Let $d_2 = 900cm = \frac{900}{100} = 9m$,

Number of spaces $n_2 = 3$

Time taken, $t_2 = n_2T = 3 \times 0.05 = 0.15s$

Final velocity, $v = \frac{d_2}{t_2} = \frac{9}{0.15} = 60ms^{-1}$

Number of spaces between mid points of PQ and RS, n = 1 + (30 – 1) + 1.5 = 31.5

$$\begin{matrix} Time\ taken \\ for\ change \end{matrix} = \begin{pmatrix} number\ of\ spaces \\ between\ mid\ points\ of \\ PQ\ and\ RS \end{pmatrix} x\ period$$

$\Leftrightarrow t = nT = 31.5 \times 0.05$

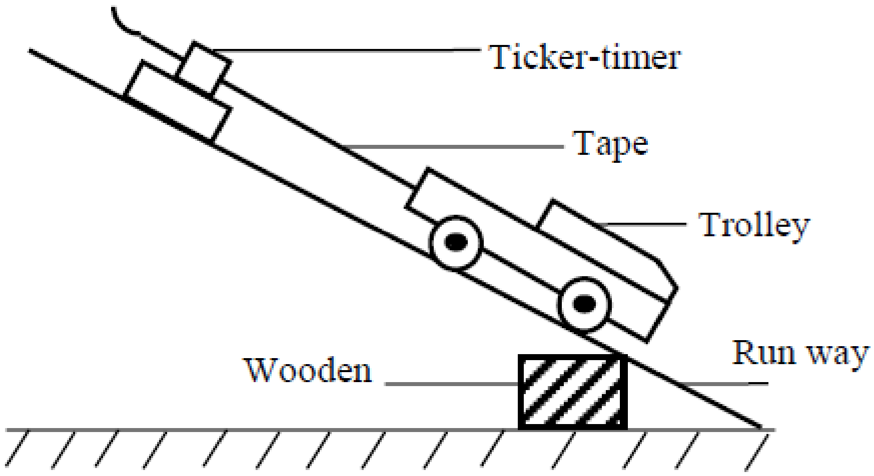
$t = 1.575s$

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken for change}}$$

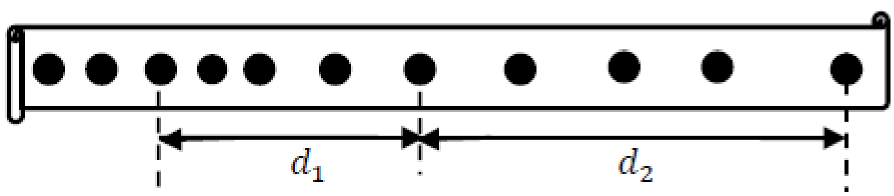
$$a = \frac{v - u}{t} = \frac{60 - 10}{1.575} = \frac{50}{1.575}$$

Thus, acceleration, $a = 31.75ms^{-2}$

DETERMINING THE VELOCITY AND ACCELERATION OF A BODY USING A TICKER TAPE TIMER:



- A paper tape is driven through a ticker timer connected to a mains supply of known frequency, f by a trolley running freely on an inclined plane.
- After the trolley has reached the end of the run way, the tape is removed and marked every after 5dots.



- The time 't' between n – spaces is calculated from; $t = number\ of\ spaces \times Periodic\ time$(one tick).
- The speed or velocity at different times is the calculated by measuring the distances $d_1\ and\ d_2$ covered in those times.
- Thus velocities; $v_1 = \frac{d_1}{t_1}$ and $v_2 = \frac{d_2}{t_2}$.
- The acceleration of the trolley is then calculated from, Acceleration $a = \frac{v_2 - v_1}{t_3}$,

Where $t_3 = nT$ is the total time taken to cover distances $d_1\ and\ d_2$,
n is the number of spaces between mid points of d_1 and d_2

OR

- The procedures are repeated, various velocities determined and a graph of velocity against time plotted. The slope of the graph gives the acceleration of the body.

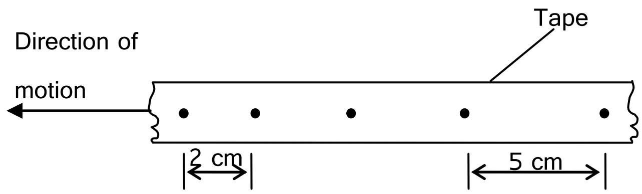
COMPENSATION FOR FRICTION

Before each experiment with a trolley, it is necessary to compensate for friction.
This can be done by tilting the runway with suitable packing pieces until it moves with uniform velocity after having been given a slight push.

Trial questions

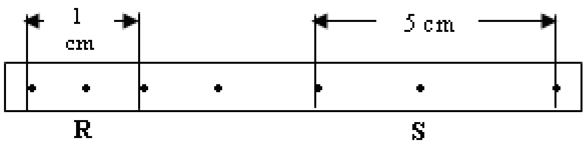
1. (a) Describe an experiment to demonstrate friction compensation using and inclined plane.

(b) The figure below shows dots produced on a tape pulled through a ticker-timer by a moving body.

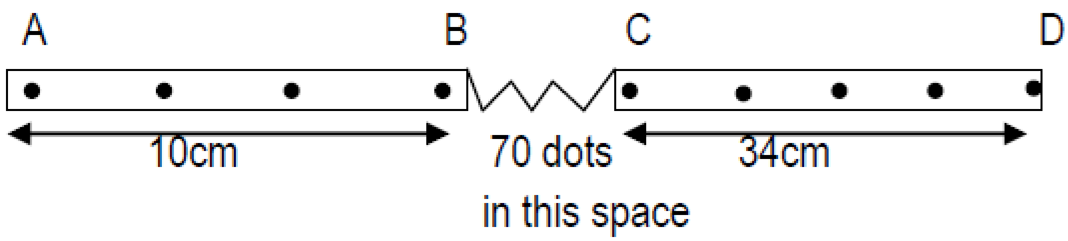


The frequency of the ticker-timer is 50 Hz. Calculate the acceleration of the body.

2. The tape shown in the diagram below was made by a trolley moving with a constant acceleration. If the frequency of the ticker-timer is 50 Hz, find the acceleration in m/s^2.



3. Below is a tape by a tickle – timer of frequency 50Hz



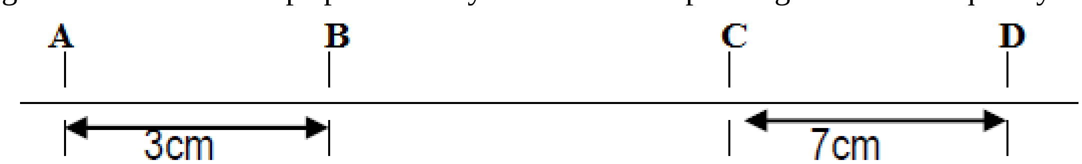
Calculate the;

(i) Initial velocity

(ii) Final velocity

(iii) acceleration of the trolley

4. The figure below shows a tape produced by a ticker timer operating at a main frequency of 50Hz



Calculate the acceleration shown by the tape

MOTION UNDER GRAVITY

ACCELERATION DUE TO GRAVITY (g)

When a body is falling down, its speed keeps on increasing per second by a constant value. This is the value of acceleration due to gravity. Its direction is towards the earth's centre or earth.

Acceleration due to gravity is the rate of change of velocity with time for a free falling body.

OR

It is the rate of change of velocity with time for a body falling towards the centre of the earth.

$g = 9.81ms^{-2}$

or

$g \propto 10ms^{-2}$

NB. A body accelerates whenever a force is acting on it. For example for a falling body, the force that acts on it is its Weight.

For a body falling down freely, its acceleration, $a = g \downarrow = 10ms^{-2}$.

Equations of motion for a body falling down freely are;

$$v = u + gt \qquad s = ut + \frac{1}{2}gt^2 \qquad v^2 = u^2 + 2gs$$

For a body thrown vertically upwards its acceleration, $a = g \uparrow = -10ms^{-2}$

Equations of motion for a body thrown vertically upwards are;

$$v = u - gt \qquad s = ut - \frac{1}{2}gt^2 \qquad v^2 = u^2 - 2gs$$

Minus because it is in a direction opposite that of a free falling body. i.e. the g is down wards and the body is moving up (direction opposite that of g).

Examples

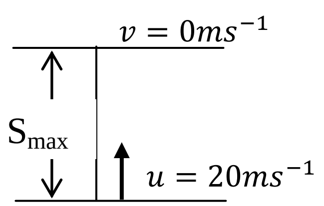
1. A ball is thrown vertically upwards with an initial speed of $20ms^{-1}$. Calculate the;

(a) Time taken to return to the thrower

(b) Maximum height reached

Solution

$u = 20ms^{-1}$, $g = -10ms^{-2}$ (since it is projected upwards)



At maximum height, $v = 0ms^{-1}$

(a) Using $v = u - gt$,

$0 = 20 - 10t$

$20 = 10t$

$t = 2s$

Time taken to reach maximum height = 2s

But the total time taken to return to the thrower = 2t

= 2 x 2

= 4s

(b) Using $v^2 = u^2 - 2gs$

$0 = 20^2 - 2 \, x \, 10 \, x \, s_{max}$

$400 = 20 \, s_{max}$

$s_{max} = 20m$

2. A stone is raised from rest at point 20m above the ground so as to fall freely vertically down wards. Find

(a) The time to land on the ground.

(b) The Velocity

Solution

u = 0m/s , s = 20m , a = g = $10m/s^2$,

(a) Using $s = ut + \frac{1}{2}gt^2$

$$20 = 0 + \frac{1}{2}\ 10\ x\ t^2$$

$\therefore Time\ t = 2s$

(b) Using $v = u + gt$

$v = 0 + 10\ x\ 2$

$\therefore Velocity\ v = 20ms^{-1}$

3. A ball is thrown vertically upwards with an initial velocity of 30m/s find the;

(a) maximum height

(b) Time taken to reach the maximum height

(c) Time taken to return to the starting point (time of flight).

Solution

u = 30m/s , a = g = $10m/s^2$,

(a) Using $V^2 = u^2 - 2gs$

0 = 900 – 2 x10 x s

= 900 -20s

∴ Maximum height S = 45m

(b) Using $v = u - gt$

$0 = 30 - 10t$

∴ Time taken to reach the maximum height, t = 3s

(c)Time taken to return to the starting point = 2t

= 2 x 3

= 6s

Exercise

1. A man stands on the edge of a cliff and throws a stone vertically upwards at $15ms^{-1}$. After what time will the stone hit the ground 20m below the point of projection

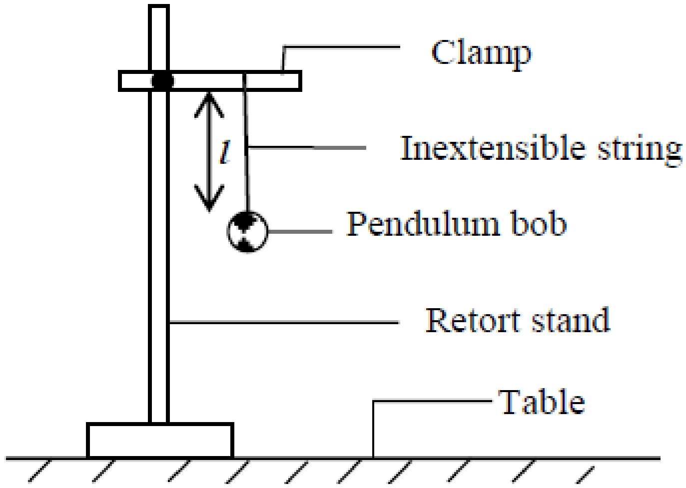
2. A stone thrown vertically upwards with an initial velocity of 14m/s neglecting air resistance, find

(a) The maximum height reached.

(b) The time taken before it reached the ground

AN EXPERIMENT TO DETERMINE THE ACCELERATION DUE TO GRAVITY

Method I: using a simple pendulum bob



Procedure:

- A pendulum bob is suspended from the retort stand using a thread of length ***l*** measured using a metre rule
- The bob is displaced through a small angle and released so that it oscillates in the vertical plane.
- The stop clock is started and the time taken to make 20 oscillations (20T) is measured and recorded.
- The period time T for a single oscillation is calculated and recorded.
- The experiment is repeated for other increasing values of *l*, and the corresponding values of 20T, T and T^2 calculated and tabulated.
- A graph of T^2 against *l* is plotted. It is a straight line graph through the origin and its slope, S is calculated.
- The acceleration due to gravity, g is then calculated from; $\boldsymbol{g = \frac{4\pi^2}{S}}$