

6.1 RATIO AND PROPORTION

How ratios are written

There are two ways of writing ratios;

If a and b are any two numbers then we can write the two numbers in any of the following ways:

(i) $a : b$ (ii) $\frac{a}{b}$

- The quantities in a ratio must be expressed in the same units.
- The order of the quantities in a ratio must be followed strictly.

- **Worked examples:**

6.1. Simplify the ratio $40 : 24$

Solution:

$$40 : 24 = \frac{40}{8} : \frac{24}{8} \text{ by dividing by } 8 \text{ on both sides}$$

$$\therefore 40 : 24 = 5 : 3$$

6.2. Simplify the ratio $10\text{cm} : 2\text{m}$

Solution:

$$1\text{m} = 100\text{cm}$$

$$2\text{m} = (2 \times 100)\text{cm} \quad : \text{ By converting m to cm}$$

$$= 200\text{cm}$$

$$\frac{10\text{cm}}{10\text{cm}} : \frac{200\text{cm}}{10\text{cm}} \quad \text{By dividing by } 10\text{cm} \text{ on both sides}$$

$$\therefore 10\text{cm} : 2\text{m} = 1:20$$

6.3. Express the ratio $2 \text{ weeks} : 8 \text{ days}$ in its simplest form

Solution:

$$1 \text{ week} = 7 \text{ days}$$

$$2 \text{ weeks} = (2 \times 7) \text{ days} \quad : \text{ By converting a week to days)}$$

$$= 14 \text{ days}$$

$$\begin{aligned} &\therefore 14 \text{ days} : 8 \text{ days} \\ &\therefore \frac{14}{2} : \frac{8}{2} \end{aligned}$$

By dividing by 2 on both sides

$$\therefore \mathbf{2 \text{ weeks} : 8 \text{ days} = 7 : 4}$$

6.4. Simplify the ratio 0.36 : 0.24

Solution:

$$0.36 : 0.24 = \frac{36}{100} : \frac{24}{100}$$

By writing 0.36 and 0.24 as fractions

$$\frac{36}{100} \times 100 : \frac{24}{100} \times 100$$

: By multiplying 100 on both sides to to eliminate the denominator.

$$\frac{36}{12} : \frac{24}{12}$$

: By Dividing by 12 on both sides

$$\therefore \mathbf{0.36 : 0.24 = 3 : 2}$$

6.5 Simplify 1 ½:6

Solution :

$$1 \frac{1}{2} : 6 = 3/2:6$$

By 1 ½ as an improper fraction.

$$3/2 \times 2 : 6 \times 2$$

: By multiplying 2 on both sides.

$$=3/3 : 12/3$$

:By dividing both sides by 3

$$1 \frac{1}{2} :6: = 1:4$$

6.6 Simplify the ratio 8:1.5

Solution:

$$8 \times 10: 1.5 \times 10$$

:By multiplying 10 on both sides to eliminate the decimal point

$$= 80/5 : 15/5$$

:By dividing 5 on both sides

$$\mathbf{8:1.5 = 16: 3}$$

6.7 The ratio of the marks scored by two students is 4.5. If the higher mark is 60, find the lower mark.

Solution:

$$\text{Lower mark : higher mark} = 4: 5$$

$$\frac{\text{Lower mark}}{\text{higher mark}} = \frac{4}{5} \quad : \text{ By writing the in fractional form.}$$

$$\frac{\text{Lower mark}}{60} = \frac{4}{5} \quad : \text{ By substituting 60 for higher mark.}$$

$$\frac{\text{Lower mark}}{60} \times 60 = \frac{4 \times 60}{5} \quad : \text{ By multiplying 60 on both sides.}$$

$$\text{Lower mark} = \frac{4}{5} \times 60 = 48$$

$$\frac{\text{Lower mark}}{60} \times \frac{60}{5} = \frac{4 \times 60}{5} \quad : \text{ By multiplying 60 on both sides.}$$

$$\text{Lower mark} = 48 \times 60 = 48$$

$$\therefore \text{ The lower mark} = 48$$

- 6.8 There are 25 Doctors in a Hospital of 450 patients. Find the Doctor to patient ration in its simplest form.

Solution:

$$\text{Doctor: patient} = \text{No. of Doctors} : \text{No. of patients.}$$

$$= 25 : 450$$

$$= \frac{25}{25} : \frac{450}{25} \quad \text{By dividing 25 on both sides}$$

$$\therefore \text{ Doctor : Patient} = 1 : 8$$

- 6.10 In certain company the ratio of men to women is 4.5. If there are 280 men. How many women are there?

Solution:

Keep the order of the quantities in the ration men : women = 4 : 5

$$\frac{\text{Men}}{\text{Women}} = \frac{4}{5} \quad : \text{ by writing the quantities in fraction form.}$$

$$280 \times 5 = 4 \times \text{women} \quad : \text{ by cross multiplying.}$$

$$\frac{1400}{4} = \frac{4 \times \text{women}}{4}$$

350 = women

∴ There are 350 women in the company.

6.11 Increase 270 in the ratio 4: 3

Solution:

New amount : original amount = 4: 3

$\frac{\text{New amount}}{\text{Original amount}} = \frac{4}{3}$ by writing the quantities in a fractional form.

$\frac{\text{New amount}}{270} = \frac{4}{3}$: by multiplying 270 for original amount.

New amount x 270 = 4 x 270 : by multiplying 270 on both sides to eliminate it from L.H.S.

New amount = 4 x 90

∴ New amount = 360

6.12. Decrease 33000 in the ration 5:6

Solution:

New amount : original amount = 5 : 6

$\frac{\text{New amount}}{\text{Original amount}} = \frac{5}{6}$: by writing the quantities in a fractional form.

$\frac{\text{New amount}}{33000} = \frac{5}{6}$: by substituting 33000 for original amount.

$\frac{\text{New amount}}{33000} = \frac{5}{6}$: by substituting 33000 on both sides to eliminate it form L.H.S

New amount = 27500

6.13. Two friends invest Shs 15,000 and Shs 30,000 in a business. At the end of the -year, they get Shs. 63,000 as profit. Find how much each get if they are to share the profits according to how much each invested.

Solution:

Let the two shares be A and B.

write the amount they invested in ration from.

A. B = 15,000 : 30,000

$\frac{15000}{15000} : \frac{30000}{15000}$: by dividing 15000 on both sides in order to simplify.

$$\therefore A : B = 1 : 2.$$

Method 1;

Now let one share be x.

A gets $X \times 1$ shares.

and B gets $X \times 2$ shares.

But total share = 63,000/=

$$\therefore X + 2X = 63,000$$

$$\frac{3X}{3} = \frac{63,000}{3} \quad \text{by dividing 3 on both sides.}$$

$$X = 21,000/=$$

\therefore A gets 21,000/= and B gets 21,000 x 2 42,000/=

Method II:

A : B = 1 : 2 from above.

Total of the quantities in ration = 1 + 2 = 3

$$\therefore \text{A's share} = \frac{1}{3} \times 63,000/=$$

$$= 21,000/=$$

and B's share = $\frac{2}{3} \times 63,000$

$$= 42,000/=.$$

- 6.14. Three people invest 90,000/=, 60,000/= in business respectively. How should they share out the profits of 90,000/=

Solution:

Let the three shares be A,B and C respectively such that A:B :C =90,000 :60,000: 30000.

$\frac{90000}{30000} : \frac{60000}{30000} : \frac{30000}{30000}$: by dividing 30000 all through to simplify.

$$A : B : C = 3 : 2 : 1$$

Method 1

Let one share be y .

A gets $3 \times y = 3y$ shares.

B gets $2 \times y = 2y$ shares.

But total of the shares = total profits

$$3y + 2y + y = 90,000$$

$$\frac{6y}{6} = \frac{90,000}{6} : \text{ by dividing both sides.}$$

$$y = 15,000/=$$

$$A \text{ gets } 3 \times 15,000 = 45,000/=$$

$$B \text{ gets } 2 \times 15,000 = 30,000/=$$

$$\text{and C gets } 1 \times 15,000 = 15,000/=$$

Method 11:

A : B : C = 3 : 2 : 1 from above.

Total of the quantities in ratio = $3 + 2 + 1 = 6$

$$\begin{aligned} A's \text{ share} &= \frac{3}{6} \times 90,000/= \\ &= 45,000/= \end{aligned}$$

$$B's \text{ share} = \frac{2}{6} \times 90,000/=$$

$$= 30,000/=$$

$$\text{And C's share} = \frac{1}{6} \times 90,000$$

$$= 15,000/=.$$

- 6.15 A sum of money is divided in two parts in the ration 4 : 7. What will be the amount if the difference between the larger and smaller share is 6,000/=.

Solution:

Method I:

Let the two people be A and B and one share be x.
 Such that A's share = 4 x X = 4x shares
 And B's share = 7 x X = 7xshare.

But large share – smaller share + 6,000

$$7x - 4x = 6000 \text{ :by substitution } 7x \text{ and } 4x \text{ for large and small shares respectively}$$

$$\frac{3x}{3} = \frac{6000}{3} \text{ :by dividing 3 on both sides.}$$

$$X = 2,000/=$$

$$= 8,000/=$$

$$\text{And B} = 7 \times 2,000$$

$$= \mathbf{14,000/=}$$

Method II:

Let the amount to be shared be m.
 Total of the quantities in the ration = 4 + 7 = 11.
 If A and B are two people sharing them

$$\text{A will get } \frac{4}{11} \times m = \frac{4m}{11} \text{ (smaller share)}$$

$$\text{But } \frac{7m}{11} - \frac{4m}{11} = 6,000$$

$$\frac{7m}{11} \times 11 = \frac{4m}{11} \times 11 = 6,000 \times 11 \text{ :by multiplying } 11 \text{ by all the terms in the express}$$

$$7m - 4m = 66,000$$

$$\frac{3m}{3} = \frac{66000}{3} \text{ : by dividing 3 on both sides}$$

$$\mathbf{m = 22,000/=}$$

$$\therefore \text{A will get } \frac{4}{11} \times 22,000$$

$$= \mathbf{8,000/=}$$

$$\text{And B will get } \frac{7}{11} \times 22,000$$

$$11$$

$$= 14,000/=$$

- 6.16. A plot of land is divided into two portions in the ration 3:8 such that the difference between the larger plot and smaller plot is 2000 hectares. Find the size of the two portions.

Solution:

- Let the size of the plot land be x hectares and the size of the portions be A and B respectively.
- Total of quantities in the ration = $3 + 8 = 11$

Such that $A = \frac{3x}{11}$ hectares.

and $B = \frac{8x}{11}$ hectares

But the larger portion – smaller portion = 2,000 hectares.

$$\frac{8x}{11} - \frac{3x}{11} = 22,000$$

$$\frac{8x}{11} \times 11 - \frac{3x}{11} \times 11 = 2,000 \times 11 \quad \text{by multiplying all the terms by 11}$$

$$5x - 3x = 22,000$$

$$\frac{5x}{5} = \frac{22,000}{5} \quad \text{:by dividing 5 on both sides.}$$

$$X = 4,400$$

$$\therefore A = \frac{3}{11} \times 4,400$$

$$= 1,200 \text{ hectares.}$$

$$\text{and } B = \frac{8}{11} \times 4,400$$

$$= 3,200 \text{ hectares.}$$

- 6.17. If $A : B = 3 : 4$ and $B : C = 1 : 2$, find $A : B : C$

Solution

- the two ratios can be combined by making the B's number the same in both ratios.
- we do this by finding the L. C.M of 4 in the first ration and 1 in the second ratio.
- ∴ L.C.M of 1 and 4 = 4
- ∴ we multiply the first ratio by 1 and the second ration by 4.
- i.e $A : B = (3 : 4) \times 1 = 3 \times 1 : 4 \times 1 = 3 : 4$ ----- (i)
- $B : C = (1 : 2) \times 4 = 1 \times 4 : 2 \times 4 = 4 : 8$ ----- (ii)
- the ratios in (i) and (ii) may now be combined to give $A : B : c = 3 : 4 : 8$

6.18. If $P : Q = 5 : 6$ and $Q : R = 4 : 7$, find $P : Q : R$.

Solution:

- the two ratios can be combined by making the Q's number the same in both ratios.
- we do this by findind the L.C. M of 6 in the first ratio and 4 in the second ratio.

2	6	4
2	3	2
3	3	1
	1	1

∴ L.C. M of 6 and 4 = $2 \times 2 \times 3 = 12$

We multiply the first ration by 2 to make a 6 equal to 12 and multiply the second ratio by 3 to make a 4 equal to 12.

$P:Q = (5:6) \times 2 = 5 \times 2 : 6 \times 2 = 10:12$ -----(i)

$Q:R = (4:7) \times 3 = 4 \times 3 : 7 \times 3 = 12:21$ -----(ii)

i.e the ratios in (i) and (ii) above may now be combined.
Hence $P : Q : R = 10 : 12 : 21$.

6.19. If $D : E = 5 : 4$ and $E : F = 2 : 3$, Find $D : E : F$.

Solution:

- the two ratios can be combined by making the E's number the same in the two rations.
- we do this by finding the L.C.M of 4 and 2.

i.e.

2	4	2
2	2	1
1	1	

∴ L.C.M of 4 and 2 = $2 \times 2 \times 2 = 4$

- we multiply the first ratio by 1 in order to retain the value of 4 for E and multiply the second ratio by 2 in order to make the 2 equal to 4.

∴ $D : E = (5 : 4) \times 1 = 5 \times 1 : 4 \times 1 = 5 : 4$ ----- (i)

and $E : F = (2 : 3) \times 2 = 2 \times 2 : 3 \times 2 = 4 : 6$ -----(ii)

the ratios in (i) and (ii) above may now be combined.
Hence $D : E : F = 5 : 4 : 6$.

6.20. If $A : B = 3 : 5$ and $B : C = 4 : 7$, find $A : B : C$.

Solution:

- the two ratios can be combined by making the B's number in the two ratios.
- We do this by finding the L.C.M of 5 in the first ratio and in the second ratio.

4	4	5
4	1	5
5	1	1

L.C.M of 4 and 5 = $4 \times 5 = 20$.

- we multiply the first ratio by 4 in order to make the B's number equal to 20 and multiply the second ratio by 5 in order to make the B's number equal to 20.

∴ $A : B = (3 : 5) \times 4 = 3 \times 4 : 5 \times 4 = 12 : 20$ (i)

and $B : C = (4 : 7) \times 5 = 4 \times 5 : 7 \times 5 = 20 : 35$ (ii)

the ratios in (i) and (ii) above may now be combined.

Miscellaneous Exercise 6.0:

1. a) 20 : 32 b) 16 : 80 c) 12 : 16 d) 12 : 28

2. a) 4 : 24 b) 25 : 15 : 30 c) 4cm : 1m d) 1:2:3

3. a) $1.25 : 4 : 3$ b) $5\frac{1}{2} : 3\frac{1}{2}$ c) $0.54 : 0.27$ d) $\frac{1}{2} : \frac{1}{8}$
4. a) $5 : 4$ b) $5 : 4$ c) $1.5 : 1.25$ d) $0.5 : 8$

Divide the following amounts in the ratio shown;

5. a) 2,000 in the ratio 3: 2 b) 600 in the ratio 1: 2 : 3
6. a) 12,000 in the ratio 1: 2 b) 0.98 in the ratio 4.4 : 6.8
7. a) 22,000 in the ratio 5 : 4 : 2 b) 24.56 in the ratio 4 : 5 : 8
8. a) 10,000 in the ratio 2 : 3 b) 4,3300 in the ratio 2 : 3 : 4
9. A certain type of concrete is made of cement power and sand in the ratio 3; 5. What weight of each would be needed to the following weights of dry mix?
 a) 24kg b) 48kg c) 800kg
10. 900kg of alloy contains 200kg of Copper and Zinc.
 a) Write the ratio of the weight of Zinc in its simplest form.
 b) What weights of Copper and Zinc would be needed for the following weights of alloy.
 (i) 5 tones (ii) 5kg (iii) 45kg
11. Three football fans win 100,000/= on the football pools. They agree to share this sum in the same ratio as their weekly stake; 2: 3: 5. How much does each receive?
12. A sum of money is divided in the ratio 3 : 4 : 7. What will be the two amounts if the difference between the larger and smaller is 92,000/=
13. A piece of land is divided in the ratio 9: 12: 13. Find the difference between the largest and smallest in 1,300 hectares.
14. If $A : B = 7 : 2$ and $B : C = 8 : 13$. find $A : B : C$
15. If $P : Q = 3 : 7$ AND $q : r = 21 : 5$. find $P : Q : R$
16. If $A : B : C = 3 : 7$ find $A : B$ and $B : C$

6.2 PROPORTION:

- Under this topic we shall look at the following:
 - (i) direct proportion
 - (ii) inverse (indirect proportion)
- **Direct proportion:**
 - In this case when one quality increases the other must also increase.
 - and if one quantity decreases, the other must also decrease.
- **Worked Examples:**

6.21 The cost of one 8 textbooks is 40,000/= .Find the cost of 6 textbooks.

Solution:

Method 1 (unitary method)

8 text books \longrightarrow 40,000/=

$\frac{8 \text{ text books}}{8} \longrightarrow \frac{40,000/=}{8}$: by dividing 8 on both sides.

= 5,000/=

1 book - 5,000/=

\therefore 6 books \longrightarrow 6 x 5,000/=

= 30,000/=

Method II

- the number of textbooks has decrease in the ration 6 : 8.
- The cost will also decrease in the ration 6 : 8

$$\begin{aligned} \text{The cost of 6 text books} &= \frac{6}{8} \times 40,000/= \\ &= 30,000/= \end{aligned}$$

6.22 A woman can walk 24km in 6 hours. How long will she take to walk 8km.

Solution :

Method 1 (unitary method)

24km \longrightarrow 6hours

$\frac{24\text{km}}{24} \longrightarrow \frac{6\text{hours}}{24}$: by dividing 24 on both sides.

$$1\text{km} \longrightarrow \frac{1}{4} \text{ hours}$$

$$\therefore 8\text{km} \longrightarrow \left(\frac{1}{4} \times 8 \right)$$

$$= 2 \text{ hours}$$

Method II

The distance has decreased in the ratio 8 : 24 or 1 : 3.
The time must also decrease in the same ratio 1 : 3

$$\therefore \text{Time taken to walk } 8\text{km} = \frac{1}{3} \times 6 \text{ hours}$$

$$= 2 \text{ hours}$$

- 6.23. A factory can produce 72 bicycles in 9 days. How many bicycles can it produce in 10 days?

Solution:

Method 1 (Unitary method)

$$9 \text{ days} \longrightarrow 72 \text{ bicycles}$$

$$\frac{9 \text{ days}}{9} \longrightarrow \frac{72 \text{ bicycles}}{9} \quad : \text{ by dividing 9 on both sides.}$$

$$1 \text{ day} \longrightarrow 8 \text{ bicycles}$$

$$\therefore 10 \text{ days} \longrightarrow (10 \times 8) \text{ bicycles}$$

$$= 80 \text{ bicycles}$$

- 6.24. A class can plant 1,000 flowers in 4 hours. How many hours can it plant in 7 hours.

Solution:

Method 1 (Unitary method)

$$4 \text{ hours} \longrightarrow 1,000 \text{ flowers}$$

$$\frac{4 \text{ hours}}{4} \longrightarrow \frac{1,000 \text{ flowers}}{4} \quad : \text{ by dividing 4 on both sides.}$$

$$1 \text{ hour} \longrightarrow 250 \text{ flowers}$$

$$\therefore 7 \text{ hours} \longrightarrow (7 \times 250) \text{ flowers}$$

= 1750 flowers.

Method II

The time has increased in the ratio 7 : 4.

The number of flowers to be planted must also be in the same ratio 7 : 4.

In 7 hours a class can plant = $\frac{7}{4} \times 1000$ flowers

= 1750 flowers.

- 6.25. A businessman took 45 million shillings to the bank. The bankers counted the money in 2 hours. How long working at the same rate would they have to take to count 60 million shillings.

Solution:

Method 1 (unitary method)

45m \longrightarrow 2 hours

$\frac{45m}{45} \longrightarrow \frac{2 \text{ hours}}{45}$: by dividing 45 on both sides.

1m $\longrightarrow \frac{2 \text{ hours}}{45}$

= $2\frac{2}{3}$ hours.

Method II

The amount of money has increased in the ratio 60 : 45 or 4:3.

The time must also increase in the same ratio 4 : 3.

\therefore Time taken to count 60m = $(\frac{4}{3} \times 2)$ hours
= $2\frac{2}{3}$ hours

- **Inverse Proportion**

In this case if one quantity increases then the other decreases.

And if one quantity decreases then the other increases.

- **Worked examples:**

- 6.26. 8 workers can dig a piece of land in 15 hours. How long will 6 workers take to dig the same piece of land?

Solution:

Method I (unitary method)

8 workers \longrightarrow 15 hours ----- (i)

1 worker \longrightarrow (8×15) hours : by dividing (i) by 8 on L.H.S and multiplying by 8 on R.H.S.

\therefore 6 workers \longrightarrow 120 hours by dividing 120 by 6

= 20hours.

Method II

The number of workers has decreased in the ratio 6:8

The time taken will increase in the ratio 8:6

Therefore 6 workers will take = $\frac{8}{6} \times 15$ (hours)

= 20hrs.

Method I

90km/hr \longrightarrow 8hrs

1km/hr \longrightarrow $\frac{90 \times 8\text{hrs}}{144}$:by dividing by 144

$$= \frac{720\text{hrs}}{144}$$

= 5hrs.

Method II

The speed has increased in the ratio 144 : 90 or 8 : 5.

The time will decrease in the ratio 5 : 8.

\therefore At 144km/hr time taken will be = $(\frac{5}{8} \times 8)$ hours

= 5 hours.

b) Method I

90 km/hr \longrightarrow 8hours

1 km/hr \longrightarrow $(\frac{90 \times 8}{80})$ hours.

$$\therefore 80 \text{ km/hr} = \frac{720}{80} \text{ hours}$$

= 9 hours.

Method II

The speed has decreased in the ratio 80 : 90 or 8 : 9.

The time will increase in the ratio 9 : 8.

The number of machines must decrease in the ratio 5: 13

$$\begin{aligned}\therefore \text{The number of machines will be} &= \left(\frac{5}{13} \times 14\right) \text{ machines} \\ &= \mathbf{5.38}\end{aligned}$$

Hence 5 machines will be required.

• **Miscellaneous worked examples**

- 6.29. 6 machines working for 8 hours a day can dig a water reservoir in 10 days. How many days would it take with:
- 4 machines working for 5 hours a day.
 - 8 machines working for 12 hours a day.

Solution:

- a) Let the number of days required be x .

$$\text{work to be done.} = 6 \times 8 \times 10 \text{ machine - hours - days}$$

$$\frac{4 \times 5 \times X}{4 \times 5} = \frac{6 \times 8 \times 10}{5 \times 5} \quad \text{:by dividing by 24 on both sides}$$

$$\mathbf{X = 24 \text{ days.}}$$

- b) let the number of days required be m

$$\text{work to be done} = 6 \times 10 \text{ machines - hours - days}$$

$$8 \times 12 \mathbf{m} = 480$$

$$\frac{96\mathbf{m}}{96} = \frac{480}{96} \quad \text{: by dividing on both sides by 96}$$

$$\mathbf{m = 5 \text{ days.}}$$

- 6.30. A building company employs 6 workers to work at the rate of 8 hours in order to complete a certain building in 10 days. Find how many workers would be required if they were to work for:
- 15 days working at a rate of 4 hours a day?
 - 16 days working at a rate of 6 hours a day?

Solution:

- a) Let the number of workers be q

$$\text{work to be done} = 6 \times 8 \times 10$$

$$\frac{60q}{60} = \frac{480}{60} \quad : \text{ by dividing by 60 on both sides}$$

$$q = 8 \text{ men}$$

b) Let the number of workers be H

$$\text{work to be done} = 6 \times 8 \times 10 \text{ men - hour - days}$$

$$H \times 6 \times 16 = 480$$

$$\frac{96H}{96} = \frac{480}{96}$$

$$H = 5 \text{ men}$$

- 6.31. A and B are two water taps such that tap A takes 6 hours to fill a certain container, tap B takes 4 hours to fill the same container, find how long would two taps take to fill the container if used together.

Solution:

The fraction of the container filled by Tap A in 1 hour = $\frac{1}{6}$ and

The fraction of the container filled by Tap B 1 hour = $\frac{1}{4}$

Both taps would take = $(1 \div \frac{5}{12})$ hours

$$= (1 \times \frac{12}{5}) \quad : \text{ by multiplying by the reciprocal of } \frac{5}{12}$$

$$= 2.4 \text{ hours}$$

- 6.32. 20 men can do a piece of work in 15 hours. How will they take if after 8 hrs they are joined by 5 other men.

Solution:

No. of men – hour required to do the piece of work.

$$= 20 \times 15$$

$$= 300$$

$$\text{No. of men – hour of work left after 8 hours} = 300 - 20 \times 8 = 140$$

$$\text{No. of hour – hours required to complete the work} = \frac{140}{20 + 8}$$

$$= 5 \text{ hours}$$

$$\therefore \text{Total No. of hours required} = 8 \text{ hours} + 5 \text{ hours}$$

$$= 13 \text{ hours.}$$

- 6.33. 10 students can dig a school flower garden in 16 hours. How long will they take if after 12 hours they are joined by 9 other students.

Solution:

$$\text{No. of student - hour required to dig the flower garden} = 10 \times 16$$

$$= 160$$

$$\text{No. of student - hour of work left after 12 hours} = 160 - 10 \times 12$$

$$= 40$$

$$\text{No. of hours required to complete the work}$$

$$= \frac{40}{10 + 9}$$

$$= 2.11 \text{ hours}$$

$$\therefore \text{Total No. of hours required}$$

$$= 12 \text{ hours} + 2.11 \text{ hours}$$

$$= 14.11 \text{ hours.}$$

- 6.34. A and B are two students such that student A takes 20 min to sweep a certain classroom and B takes 25 min to sweep the same class. How long will they take to sweep the class both working together?

Solution:

$$\text{In one minute A sweeps} = \frac{1}{20} \text{ of the class.}$$

$$\text{In one minute B sweeps} = \frac{1}{25} \text{ of the class}$$

$$\text{In one minute both sweep} = \left(\frac{1}{20} + \frac{1}{25} \right) \text{ of the class.}$$

$$= \frac{9}{100} \text{ of the class}$$

$$\therefore \text{The no. of minutes required if they if they are to sweep together} = \left(1 \div \frac{9}{100} \right) \text{ hours}$$

$$= \left(1 \times \frac{100}{9} \right) \text{ hrs} : \text{ by multiplying by the reciprocal of } \frac{9}{100}$$

= **11.11 minutes.**

- 6.35. The map of a school is drawn to the scale of 1 : x. On this map a sports ground is represented by a rectangle measuring 3.75cm long and 1.2 cm wide. Find the value of x if the actual area of the sports ground is 36km².

Solution:

$$\begin{aligned}\text{Area of sports ground on the map} &= 3.75 \times 1.2 \\ &= 4.5\text{cm}^2\end{aligned}$$

But 1 km = 100,000cm : by converting cm to km.

$$\therefore 1 \text{ km}^2 = 1 \times 10^5 \text{ cm}^2.$$

$$\begin{aligned}4.5 \text{ cm}^2 &= \frac{4.5\text{cm}^2}{1 \times 10^5 \text{ cm}^2} \times 1\text{km}^2 \\ &= 4.5 \text{ cm}^2 = \frac{4.5 \times 100}{1 \times 10^5 \text{ cm}^2} \\ &= \frac{4.5 \times 10^0 \text{ cm}^2}{1 \times 10^5} \\ &= \mathbf{4.5 \times 10^{-5}\text{km}^2}\end{aligned}$$

$$\text{Actual area sport ground} = 36\text{km}^2$$

$$\begin{aligned}\therefore \text{Area scale factor} &= \frac{\text{Actual area on ground.}}{\text{Area on the map.}} \\ &= \frac{36}{4.5 \times 10^{-5}} \\ &= \mathbf{8 \times 10^5}\end{aligned}$$

$$\text{A.S.F} = (\text{L.S.F})^2 \quad \text{:by formula}$$

$$\text{L.S.F} = 8 \times 10^5$$

$$\text{Hence } x = \mathbf{894.4272}$$

- 6.36. Moses left Shs. 22,050,00 million in his will to be shared between his wife and, daughter and son in the ratio 2:3:4. His wife decided to divide her share equally between her daughter and son.

Determine how much finally did the son and daughter get.

Solution:

$$\text{Total number of shares} = 2 + 3 + 4$$

$$= 9$$

$$\text{Wife's share} = \frac{2}{9} \times 22,050,000/=$$

$$\text{Wife's share divided by 2} = \frac{4,900,000}{2} =$$

$$= 2,450,000/=$$

$$\text{Daughter's share} = 2,450,000 + \frac{3}{9} \times 22,050,000$$

$$= 2,450,000 + 7,350,000$$

$$= 9,800,000/=$$

$$\text{Son's share} = 2,450,000 + \frac{4}{9} \times 22,050,000$$

$$= 2,450,000 + 9,800,000$$

$$= 12,250,000/=$$

- 6.37. Two quantities A and B are such B varies inversely as the square of A. Given that B = 16 when a = 4, Find the value of A when B = 9.

Solution:

$$B \propto \frac{1}{A^2} \text{ (This is the statement B varies inversely as the square of A)}$$

\propto = the symbol of proportionality.

$$B = \frac{1}{A^2} \times K$$

:By multiplying K the constant on the R.H.S to eliminate the symbol of proportionality

$$B = 16 \text{ when } A = 4$$

$$\therefore 16 = \frac{K}{4^2} \quad \text{: by substituting 16 and 4 for B and A respectively}$$

$$16 \times 16 = \frac{K \times 16}{16} \quad \text{: by eliminating 16 on both sides to eliminate from R.H.S}$$

$$K = 256$$

$$\text{Hence } B = \underline{256}$$

$$A^2$$

$$\text{When } B = 9, \therefore P = \frac{256}{A^2}$$

$$9A^2 = \frac{256}{A^2} \times A^2 \quad : \text{ by multiplying } A^2 \text{ on both sides in order to eliminate it from the R.H.S}$$

$$\frac{9A^2}{9} = \frac{256}{9} \quad \text{by dividing by 9 on both sides.}$$

$$A^2 = 28.4$$
$$\sqrt{A^2} = \sqrt{28.4} \quad : \text{ by taking square root on both sides.}$$

$$\mathbf{A = 5.3}$$

MISCELLANEOUS EXERCISE 6.1

1. An athlete can run 16km in 8min. Find how long would it would take to run:
(a) 8 km (b) 32km (c) 40km
2. A student carried out an experiment in a Chemistry lab and found out that 12kg of a certain chemical produced 180cm of gas. How much gas will be produced by the following weights:
3. A taxi takes 6 hours to make a journey at 70mk/h. Find how long would it take at the following speeds:
(a) 60km/h (64km/h (c) 32km/h
4. A businessman can pay his electricity bill by paying 15,000/= per week for 6 weeks. How long would he take to pay if he increased his weekly payments by:
(a) 400 (b) 64km/h (c) 32/h
5. A group of 10 workers working at a rate of 6 hours a day can slash a certain compound in 4 days. Find how many days it would take with:
(a) 8 workers working for 3 hours a day.

(b) 2 workers working for 2 hours a day.
6. Three water taps P,Q and R are such tap P takes 5 hours to fill a certain tank, tap Q takes 3 hours and tap R takes 7 hours to fill the same tank. Find how long it would take all the three taps to fill the tank if used together.
7. 5 men wish to prepare a field for Athletics competitions and they can do this in 3 hours. How long will they take if after $\frac{1}{2}$

8. 10 workers can organize the sitting arrangement in a certain hall where a big conference is to take place for 4 hours. How long will then take if hour they are joined by 6 other workers.
9. Two outlets M and N can empty a water tank in 18min and 24min respectively. How long will it take if both are used together?.
10. Two sisters one aged 18 years and the other 10 years do a certain piece of work together for 5 hours. The elder sister working along takes 9 hours to complete the work. How long will the younger one take when she works alone?