

**P425/1**

**PURE MATHEMATICS**

**Paper 1**

**Sept/Oct. 2019**

**3 hours**

**POST MOCK EXAMINATIONS 2019**

**Uganda Advanced Certificate of Education**

**PURE MATHEMATICS**

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**3 hours**

**INSTRUCTIONS TO CANDIDATES:**

*Answer **all** the **eight** questions in section A and any **five** from section B.*

*Any additional question(s) answered will **not** be marked.*

*All necessary working **must** be shown clearly.*

*Begin each answer on a fresh page.*

*Silent non programmable scientific calculators and mathematical tables with a list of formulae may be used.*

**TURN OVER**

## SECTION A: (40 MARKS)

Answer *all* questions in this section.

1. Solve the inequality:  $\frac{x+3}{x-1} \geq 2$ . (04 marks)
2. Find the angle between the line  $\mathbf{r} = 3\mathbf{k} + t(7\mathbf{i} - \mathbf{j} + 4\mathbf{k})$  and the plane  $\mathbf{r} \cdot (2\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}) = 8$ . (05 marks)
3. Use binomial series to expand  $\sqrt{\frac{1+x}{1-x}}$  up to the third term. Hence using  $x = \frac{1}{9}$ , show that  $\sqrt{5} \approx \frac{181}{81}$ . (05 marks)
4. Find and classify all the stationary points of the curve  $y = x^2 e^x$ . (05 marks)
5. Given that A and B are reflex angles such that  $\cos A = \frac{5}{13}$  and  $nB = \frac{3}{4}$ .  
Without using tables or calculator, find  $\sin(A + B)$  in fraction form. (05 marks)
6. Express  $5 + 4x - x^2$  in the form  $a - b(x - c)^2$ . Hence or otherwise, find  $\int \frac{1}{\sqrt{5+4x-x^2}} dx$ . (05 marks)
7. A chord PQ of a parabola  $y^2 = 4x$  subtends a right angle at the vertex. Show that the equation of locus of the midpoint of the chord PQ is  $y^2 = 2(x - 4)$ . (05 marks)
8. The area enclosed between the curve  $y = 4 - x^2$  and the line  $y = 4 - 2x$  is rotated through  $180^\circ$  about the x-axis. Find the volume of the solid generated. (05 marks)

## SECTION B: (60 MARKS)

Attempt only **five** questions in this section.

9. (a) Solve the equation  $\sqrt{x-1} + 2\sqrt{x-4} = 4$ . (04 marks)
- (b) Show that  $\log_a b = \frac{\log_c b}{\log_c a}$ . Hence find without using tables or calculator, the value of  $\log_{16} 128$ . (05 marks)
- (c) Solve:  $(0.6)^{x+3} = (0.4)^{2x-1}$  correct to 4s.f (03 marks)
- 10.(a) Given that  $W = -3 + 4i, Z = -24 - 7i$ , express  $V = \frac{W^3}{Z}$  in polar form. Hence or otherwise, represent  $V, W$  and  $Z$  on the same Argand diagram. (08 marks)
- (b) Find the square roots of  $-16i$ . Leave your roots in surd form (04 marks)
11. Express  $f(x) = \frac{4x^2}{(x-3)(x^2+3)}$  in partial fractions. Hence find
- (a)  $f'(-1)$  (09 marks)
- (b)  $\int f(x)dx$  (03 marks)
- 12.(a) Prove that in any triangle ABC,  $\frac{a+b-c}{a-b+c} = \tan \frac{B}{2} \cot \frac{C}{2}$ . Hence or otherwise, find angle A given  $a = 4, b = 5$  and  $c = 6$ . (07 marks)
- (b) Solve the equation  $\cos 4\theta + 2\cos^2 \theta = 1$  for  $0 \leq \theta \leq \pi$ . (05 marks)
13. A line  $l_1$  passes through point A with position vector  $5\mathbf{i} + 3\mathbf{j}$  and a point B with position vector  $-2\mathbf{i} - 4\mathbf{j} + 7\mathbf{k}$ . A line  $l_2$  has equation  $\mathbf{r} = (\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}) + \lambda(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$
- (a) Show that  $l_1$  and  $l_2$  are perpendicular. (03 marks)
- (b) C is the foot of the perpendicular from point  $D(5,3,0)$  to the line  $l_2$ . Find the coordinates of C. (06 marks)
- (c) Obtain the Cartesian equation of the plane that passes through the point C and is parallel to  $l_1$ . (03 marks)

14. A container in the shape of a hollow cone of semi vertical angle  $30^\circ$  is held with its vertex pointing downwards. Water is poured into the cone at a rate of  $20\pi\text{cm}^2\text{s}^{-1}$ . If depth of the water is 10cm, find the rate at which

(i) The depth of water is rising (04 marks)

(ii) The surface of water is increasing (03 marks)

(b) An error of 12% was made in measuring the volume of a sphere. Find the percentage error made in measuring

(i) the radius of the sphere. (03 marks)

(ii) the surface area of the sphere (02 marks)

15. Given that the point  $P\left(4t, \frac{4}{t}\right)$  lies on the curve  $xy = 16$ ,

(a) Find the equation of tangent and normal to the curve at point P.

(b) The normal at P meets the curve again at Q and the tangent at P meets the y-axis at R (05 marks)

(i) Obtain the coordinates of Q (03 marks)

(ii) Find the Cartesian equation of locus of M, the midpoint of PR. (04 marks)

16. (a) Solve the differential equation  $x \frac{dy}{dx} + 2y = \ln x$ . (04 marks)

(b) A rumour spreads at an Damba island at a rate proportional to the product of the population P that has heard the rumour and that yet to hear it. The total population of the population is 10,000. Initially the population that had heard the rumour was 1000. After a day, 2000 people had heard the rumour. Find the population that had heard the rumour by the eleventh day. (08 marks)

**END**